

Selected Departures from Published Arguments

Notes on the formalization of the classification
of finite simple groups

Axiom Math

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The classification of finite simple groups is a correct theorem and one of the great collective achievements of mathematics. This formalization rests on the ideas, methods, and arguments developed by the mathematicians who established it. The departures described here concern particular supporting statements and proof steps. They do not call the classification into question.

This document was generated by the AI system AxiomProver. The list was assembled by auditing Axiom Math's Lean formalization and comparing its arguments with the literature identified on the [project website](#). AxiomProver autonomously produced all the corrections, supplementary arguments, and alternative proofs described here in the course of the formalization. Some corrections were already known, and their inclusion makes no claim of priority.

The 96 entries record selected mathematical departures and elaborations. They are not a tally of 96 errors or new discoveries. This is not a comprehensive errata list. The arguments recorded here may not be the simplest or most natural resolutions. We welcome corrections, references to earlier treatments, and suggestions for simpler arguments or further mathematical study.

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1 Introduction

This collection accompanies Axiom Math’s formalization of the classification of finite simple groups (CFSG) from the first-generation proof and its supporting literature. It records selected points at which the formalization used a revised statement, a supplementary argument, or a different proof. It is intended for mathematicians reading the cited sources.

How to read the entries. The 96 entries are an editorial selection of mathematical departures, not a count of independent errors or new discoveries. Some give a counterexample to a particular auxiliary assertion or an omitted case in a stated list. Others supply an additional argument or replace a proof step while retaining the intended conclusion. A local example may invalidate an intermediate inference without satisfying all the hypotheses of the surrounding theorem. Section 2 identifies the relevant statement corrections.

Selection and organization. The selection concerns material changes to hypotheses, conclusions, or arguments. It is not a comprehensive errata list. Routine typographical corrections, notational changes, and readily repaired slips are omitted. Related consequences of one difficulty are grouped where appropriate. Part A contains 34 entries concerning structural changes and alternative proofs. Part B contains 62 entries concerning additional arguments and local corrections. These categories describe the extent of the change to the argument, rather than a measure of error or mathematical importance.

AxiomProver’s contribution and scope. AxiomProver produced all the corrections, supplementary arguments, and alternative proofs recorded here during the formalization. This attribution does not assert priority for every correction or underlying idea. Some were already known. Each entry specifies the result obtained. In some cases a more restricted statement suffices for the classification, and the stronger printed assertion is not recovered in its original generality.

Purpose and comments. We offer this account with respect for the mathematicians whose work established the classification. Our aims are to clarify the literature, record corrections where needed, and make the alternative arguments available for further study. We do not claim that the arguments used by AxiomProver are always the shortest, simplest, or most natural. Experts may find more direct proofs, better formulations, or useful generalizations. We welcome comments, corrections, references to earlier treatments, and suggestions for further mathematical investigation.

The Unbalanced Group Theorem. The public report [19] gives a detailed account of the closure issue in Walter’s memoir and an alternative argument for the simple-group UGT reduction. Case 1 summarizes this departure and its resolution.

Sources and notation. References identify the relevant statement and printed pages where available. Descriptions are paraphrases. Findings concerning an author-manuscript version are identified as such. All groups are finite unless an algebraic group is explicitly mentioned. Write $O(X) = O_{2'}(X)$ for the odd core, $E(X)$ for the ordinary layer, and $F^*(X)$ for the generalized Fitting subgroup. A component is a quasisimple subnormal subgroup. The term 2-component follows the cited source, usually a perfect subnormal subgroup with quasisimple quotient by its odd core. The notation $m_p(X)$ denotes elementary abelian p -rank. Other notation is local to an entry.

2 Selected corrections to statements

The entries below record counterexamples to particular assertions, or additional cases in printed lists. The assertion at issue and the scope of the example are specified in the detailed entry. A case may concern several related statements. The entries are grouped by source and ordered alphabetically by first author. This is a selection of corrections, rather than a comprehensive assessment of the cited works. The counterexamples concern supporting statements or proof steps and do not contradict CFSG. Cases 38 and 39 concern the author-manuscript versions identified in those entries.

- **Alperin, Brauer, Gorenstein, *Finite groups with quasi-dihedral and wreathed Sylow 2-subgroups* (1970), Chapter II, Section 7, Theorem 2 (the relative ZJ theorem).** Case 26. The theorem extends the Sylow-subgroup ZJ argument to a subgroup R satisfying the printed conditions. Counterexample: $G = H \wr C_2$ and $R = S \times O_p(H)$, with S Sylow in a suitable p -stable constrained group H and $Z(J(S)) \neq Z(J(O_p(H)))$. The printed conditions on R hold, but the factor-swapping element does not normalize $Z(J(R))$, so the claimed factorization fails. Correction: a relative theorem with an explicit Frattini condition on the normal subgroups met by the induction; the unrestricted statement is not needed in the application.
- **Aschbacher, *Weak closure in finite groups of even characteristic* (1981), Theorem 9.3 and (9.13).** Case 40. Counterexample: $(U \oplus U) \rtimes S_5$, with U the four-dimensional even-weight binary module. The relevant rank-three TI-subgroups give the omitted degree-two case; both the normalizer-containment and the generation alternatives of Theorem 9.3 fail, and the same model refutes (9.13) as stated. Correction: the version used adds that T is Sylow in G and that $N_G(T) \leq N_G(V)$; a Frattini argument then excludes the degree-two case. The false general theorem is bypassed, not reproved.
- **Aschbacher, *A characterization of some finite groups of characteristic 3* (1982).** Three cases.
 - Lemma 5.16(2), together with an assertion in 5.18(1). Case 20. In the natural $PSO_8^+(3)$ model an order-four rotation of an anisotropic plane centralizes the designated $\Omega_6^-(3)$ component and, having determinant one on the complementary four-space, preserves the two $SL_2(3)$ factors rather than interchanging them as claimed. Correction: the swap clause is discarded, 5.18(1) is not used, and 5.18(2)–5.20 are reproved by a different chain. The entry records that these nonsimple models refute the local deductions, not the final theorem.
 - Lemma 2.14(2), final clause. Case 83. The joint centralizer condition does not force involution type $i(2, -)$: in the seven-dimensional orthogonal model type $i(6, -)$ also occurs, and its five-dimensional common eigenspace still supplies the $\Omega_5(3)$ component. Lemma 4.5 invokes the false exclusion. Correction: 4.7 is proved first, the type exclusion is removed, and simplicity and fusion are used where the local geometry was insufficient.
 - Lemma 3.1, implications (2) $\Rightarrow$ (1) and (3) $\Rightarrow$ (1). Case 96. In $PSp_4(q)$, $q \geq 5$ odd, a factor of the central product of the two $SL_2(q)$ plane stabilizers is a component at the central involution, and its centralizer contains only that involution, yet it is not standard: the other factor is a distinct commuting conjugate. Correction: the criterion assumes explicitly that the component commutes with no distinct conjugate, and Lemma 3.3 is reassembled from the corrected criterion.
- **Aschbacher, Segev, *Extending morphisms of groups and graphs* (1992), (8.13)(2).** Case 9. For a pair whose product lies in Monster class $3C$, the common centralizer involves the Thompson group and need not contain the required $2A$ involution, so the common-neighbour assertion is false in that case; the $2A$ -pure S_4 configuration exists, so the branch cannot be excluded. In (8.15) the $4B$ argument also needs exact commutation where only commutation modulo a central involution is available. Correction: a different statement is proved, that

any two length-three paths between the relevant $3C$ pair are homotopic, and the four-cycle argument is reorganized around it.

- **Aschbacher, Seitz, *Involutions in Chevalley groups over fields of even order* (1976), (11.13)–(11.14), symplectic case.** Case 11. The unipotent group T in the relevant centralizer is abelian, so a nontrivial class in $H^1(L, T)$ gives a shear automorphism of $T \rtimes L$; in $Sp_4(4)$ such an automorphism sends a transvection in the complement to an involution of rank two. The assertion that automorphisms of the centralizer preserve the transvection class is therefore false. Correction: the replacement chooses an odd-order element in $A \cap A^g$ and uses the valid core of (7.5); the rigidity assertion is not used.
- **Aschbacher, Smith, *The classification of quasithin groups*, Volumes I–II (2004).** Seven cases.
 - Theorem C.1.34(4). Case 5. The containment $C_Q(L) \leq Z(G_0)$ fails in a verified $F_4(2)$ parabolic configuration: a long-root subgroup lies in $C_Q(L)$ but has a nontrivial commutator with another root subgroup of the Sylow 2-subgroup. Correction: the clause is deleted; the application in Chapter 15, (15.3.23), is closed by an additional global exclusion applied to the pair (LS, S) .
 - Lemma E.2.2. Case 6. The equality $n(H) = n$ fails for nonsplit configurations such as $SL_2(5)$ and $2^4:SL_2(5)$ with the natural $L_2(4)$ action, where the local parameter is one and the field parameter two; F.7.13(2) and several Chapter 6 lemmas inherit the error. Correction: $n(H) \leq n$ is retained, the case division in Chapters 5–6 is reorganized by the actual field size, and each later application establishes any required lower bound from its own hypotheses.
 - Lemma 15.3.68(2). Case 25. The equality $Q_1 \cap Q_2 = R$ fails; the local model $2^6:(L_3(2) \times 2)$ in a parabolic of $L_5(2)$, with the natural-plus-dual module, violates the conclusion. Correction: part (2) is discarded, and the closing argument of Theorem 15.3.1 uses 15.3.50 and the corrected C.1.34 instead.
 - Proposition 1.3.4. Case 51. The catalogue for an elementary abelian subgroup of order nine omits the Held group: $\text{Aut}(He)$ has the required local configuration inside a subgroup of shape $2^{4+4}:(S_3 \times S_3).2$. Correction: the Held row is added and excluded in Chapters 10 and 15 by separate arguments.
 - Theorem G.11.2, with Lemmas G.10.15(3) and G.10.16(4). Case 52. The S_8 row and the six-dimensional $U_3(3)$ row are missing: $A_8 \cong GL_4(2)$ can induce the full linear group on the relevant section, and $U_3(3)$ has admissible subspaces in two orbits of length four. Correction: both rows are added and the later arguments analyze the expanded list.
 - Lemma B.4.14, used in Theorem 4.3.2. Case 53. The module list omits the determinant twists of the sixteen-dimensional adjoint module for $L_3(4)$: modules with the same restriction to the derived group but a different action of the diagonal extension. Correction: the classification records the determinant parameter $k \in \mathbb{Z}/3\mathbb{Z}$, and each application determines whether the nontrivial twists are allowed.
 - Theorem F.6.18, cases (6), (8) and (9). Case 55. The amalgam list excludes actual amalgams supplied by $L_2(23)$ and by field extensions such as $P\Sigma L_2(25)$. Correction: the list is rederived from the Goldschmidt amalgam classification with the missing linear case and the field-extension alternatives added, and the Chapter 15 inspection is rebuilt over it.
- **Devillers, Gramlich, Mühlherr, *The sphericity of the complex of non-degenerate subspaces* (author version arXiv:0711.3468v3; published 2009), Section 4, Lemmas 4.2–4.3 and Theorem 4.6, identity-involution branch in characteristic two.** Case 39. Over $\mathbb{F}_8$ with the standard symmetric form on a three-dimensional space, the nonsingular vector $(1, 1, 1)$ has an alternating perpendicular space; the corresponding point lies in no nondegenerate plane and is isolated although the field bound is satisfied, so the nonempty-link and sphericity assertions fail in this branch. Correction: the classification route uses only the

genuine Hermitian case, with a nontrivial involution as an explicit hypothesis. No repair of the symmetric characteristic-two theorem in its original form is claimed.

- **Fritz, *On centralizers of involutions with components of 2-rank two, II* (1977), Lemma 2.6(a).** Case 43. The fixed-subgroup conditions on two commuting central automorphisms do not determine the special group of order 256: several nonisomorphic groups, distinguishable by their involution counts, satisfy them, and only one is the claimed central product of quaternion groups. Correction: a finite classification of the semidirect products $P \rtimes V$ with $P \cong UT_3(\mathbb{F}_4)$, verified by finite computation, supplies the exclusion that 3.19 needs.
- **Gilman, Griess, *Finite groups with standard components of Lie type over fields of characteristic two* (1983).** Two cases.
 - Lemma 5.11, $A_5(2)$ half-end-node subcase. Case 10. Inside $E_6(2)$ two root-centralizer components $K_1, K_2 \cong SL_6(2)$ can meet in the prescribed $SL_4(2)$ with joined reflection action $W(B_4)$ while $\langle K_1, K_2 \rangle = E_6(2)$, an outcome the printed list does not allow. Correction: the exceptional completion is retained using $E_6(2)$ gluing data, and the classical argument is used only with its automizer-containment condition. The entry records that this does not refute the main theorem that the resulting group is of Lie type.
 - Lemma 2.44, last assertion. Case 85. In the A_1A_5 subsystem of $E_6(2)$, with z of order three in the A_1 factor and y the order-three scalar on the $\mathbb{F}_4^3$ realization of the six-dimensional module, $D = \langle z, y \rangle$ cannot be conjugated into $J = SL_3(4)$: z has algebraic centralizer dimension 36, while order-three elements of J have centralizer dimension 24 or 30. Correction: the $C_G(J)$ containment needed in 6.6(iii) is proved from $E_6(2)$ model data; the conjugacy is not used.
- **Gorenstein, Lyons, *The local structure of finite groups of characteristic 2 type* (1983), Part I, (20-7)(1).** Case 44. For $G_2(8)$ at $p = 3$, an $SU_3(8)$ component contains a torus of order 57 and an element of order three normalizing its subgroup of order 19; together with the central scalar this is an elementary abelian actor of rank two normalizing a nontrivial $3'$ -subgroup, contradicting the printed balance conclusion. The proof's claim that the unitary group involves only the primes two and three holds for field size two, not eight. Correction: an additional rank-three condition on the induced actor in the $G_2(8)$ case; other ambient groups are treated by order, since $19 \nmid |G|$ excludes the component.
- **Griess, Solomon, *Finite groups with unbalancing 2-components of $\{\hat{L}_3(4), \text{He}\}$ -type* (1979), Corollary 2.14(1), with its use in (3).** Case 17. The corollary asserts normality of the designated component in the ambient group. Counterexample: $Q \wr C_2$ with Q a central 2^2 -cover of $L_3(4)$. The factor $Q \times 1$ occurs in the appropriate involution centralizer, the swapping involution moves it, the odd core is trivial, and the Held outcome is excluded by order. Correction: only the intersection statement needed by the pump-up argument, $P \cap Z(Q) = 1$, is proved.
- **Guralnick, Malle, *Classification of $2F$ -modules, II* (2004), Corollary 2.11(b),(c) and equation (2.1), inspected author manuscript.** Case 38. With $n = \dim V$ and $z = \dim V^G$, the bound $\dim(\text{Sym}^2 V)^G \leq z(n+1)/2$ fails for $G = C_3^2$ acting over $\mathbb{F}_3$ on the square of the augmentation ideal of $\mathbb{F}_3[x, y]/(x^3, y^3)$: $n = 6$, $z = 1$, and the fixed space in the symmetric square has dimension four. The exterior-square bound and the general codimension forms fail likewise. Correction: for elementary abelian groups in odd characteristic only the binomial bound $\dim(\text{Sym}^2 V)^G \leq \binom{n-z}{2} + \binom{z+1}{2}$ and its exterior-square analogue are stated; the stronger terms are proved for cyclic actors only, with that hypothesis explicit.
- **McBride, *Near solvable signalizer functors on finite groups* (1982).** Two cases.
 - Theorem 3.6(b). Case 41. The hypothesis gives only $X_2 \cap D \leq Y$, but conclusion (b) asserts equality and identifies (X_1, X_2) as the distinguished chief-factor pair. Counterexample:

- $PSL_2(32)$ with its field automorphism of order five, with a trivial actor on C_7^2 adjoined; the simple chief factor has bottom group one while its designated centralizer contains C_7^2 , and both printed alternatives fail. Correction: the alternative is restated for the pair (X_1Y, X_2Y) and the chief-factor comparison theorem is rebuilt from it.
- Theorem 2.8(d), $U_3(2^p)$ case. Case 42. The subgroup $C_1 \cong 3^2:2$ lies in both the small normalizer of 3^2 and the monomial subgroup, so the claimed unique maximal overgroup of C_1 is false; the full fixed subgroup $C \cong 3^2:Q_8$ does have the uniqueness. Correction: uniqueness is stated for C , and the use of C_1 in 6.2(d) is replaced by a prime-overgroup theorem.
 - **Meierfrankenfeld, Stellmacher, *Pushing up weak BN-pairs of rank two* (1993), (3.3)(i).** Case 45. The point and hyperplane parabolics of $L_4(q)$ form a critical pair at distance two in the paper's own sense, and the displayed quotient $Z_\alpha Q_{\alpha'}/Q_{\alpha'}$ has order q^2 , not q as asserted; already $L_4(2)$ gives index four. Correction: the index clause is deleted, the distance-two exceptional configurations are separated, (3.6) is obtained from corrected auxiliary alternatives, and (4.3) is proved without the clause.
 - **Solomon, *Finite groups with intrinsic 2-components of type $\hat{A}_n$* (1975), Theorem 4.1(3).** Case 95. When the permutation involution moves eight points as four transpositions, its centralizer has two normal regular quaternion subgroups with that involution as their unique involution; these are omitted from the normal-subgroup clause, which Lemma 5.7 uses in exactly the range $8 \leq n \leq 11$. Correction: the clause is proved only in the range where the permutation action eliminates the exception, and the four-transposition range is settled by a finite check supplied to Lemma 5.7.
 - **Solomon, *Standard components of alternating type, II* (1977), Lemma 4.7.** Case 69. The list of overgroups in $GL_6(2)$ omits $L_3(2) \times S_3$, which occurs in the relevant six-dimensional representation and is visible in a parabolic of $L_5(2)$. Correction: a complete overgroup census retaining the product case, together with a separate Sylow-order bound (a 2-part of at least 2^{11} in $N_G(T)$) that removes the small candidates.
 - **Solomon, *2-Signalizers in finite groups of alternating type* (1978), Theorem 2.6.** Case 64. The theorem is invoked to conclude $H = G$ from centralizer and fusion control. Its asserted form is too strong: $A_4 < A_5$ shows that such control can yield a proper strongly embedded subgroup, and what the cited argument establishes is strong embedding. Correction: the $m = n + 4$ case of Lemma 3.6 is closed by proving the centralizer containment for the actual involution, applying the strongly-embedded-subgroup theorem, and deriving the contradiction from an incompatible involution class.
 - **Solomon, Wong, *On $L_2(2^n)$ -blocks* (1981).** Two cases.
 - Lemma 2.16, parts (5) and (8), through the nonsplit case in the proof of (2). Case 7. For $n = 2$ the proof claims that W is elementary abelian and splits as $V \oplus Z$. There are nonsplit blocks with $X/V \cong SL_2(5)$, of orders 1920 and 3840 with $W \cong C_5^2$ and $W \cong C_4 \times C_2^4$, for which W/V has no invariant complement, and parts (5) and (8) fail for them. Correction: the split and nonsplit cases are separated, and the nonsplit configuration is excluded in the application under the stronger ambient hypotheses; the examples remain valid for arbitrary blocks.
 - Theorem 2.4, the quoted classification of characteristic-2-type groups with class-two Sylow 2-subgroups and the stated rank condition. Case 71. As printed it also admits $Sz(32)$ and $U_3(16)$, which are not on its list. Correction: an application-specific bypass through the pushing-up route and Gomi's local analysis. No proof of the printed list is claimed, nor that $Sz(32)$ or $U_3(16)$ fail the printed hypotheses.
 - **Timmesfeld, *A note on 2-groups of $GF(2^n)$ -type* (1979), Section 1: Lemma 1.2, Corollary 1.3 and Theorem 1.6.** Case 2. The abstract theory infers field multiplication

from partition and centralizer properties of certain 2-groups, but those conditions admit non-Desarguesian spread data. An explicit group of order 4096 on V^3 , $V = \mathbb{F}_2^4$, with multiplication $(u, w, z)(u', w', z') = (u + u', w + w', z + z' + \beta(u, w'))$ for a nonsingular bilinear pairing β from such a spread, has its two distinguished elementary abelian subgroups of order 256 generating it, yet a mixed-element centralizer has the wrong structure for the asserted $GF(16)$ model. Correction: the field structure is obtained from the additional local hypotheses of the Main Theorem, through (2.18) of Timmesfeld's earlier paper in Math. Z. 161 (1978); the abstract lemma is not used.

- **Timmesfeld, *Correction to “Finite simple groups in which the generalized Fitting group of the centralizer of some involution is extraspecial”* (1979), the corrected (3.11)(1).** Case 30. The corrected fixed-space equality $C_{\tilde{Q}}(\bar{L}) = \langle \tilde{a} \rangle$ is asserted uniformly in the width; it is false at width three, and at width six the assertion that the designated element is the only element of its type is incompatible with the remaining order-four possibility for D . Correction: weak closure and uniqueness of the singular vector are proved for width at least three, the equality is asserted only where the dimension argument proves it, the small widths are handled separately, and the fusion step is supplied by Timmesfeld's weakly-closed transitive-involution theorem with its hypotheses verified.
- **Walter, *The $\mathfrak{B}$ -conjecture; characterization of Chevalley groups* (1986).** Two cases.
 - Theorem I, with the definition of $\mathcal{C}_0$. Case 35. The theorem asserts that a group X with a unique 2-component and $O(C_X(E)) \not\leq O(X)$ for some nontrivial elementary abelian 2-subgroup E has its component in $\mathcal{C}_0$; the definition excludes $PSL_4(3)$ and $PSU_4(3)$. Counterexamples: $X = L_4(3).2$ and $X = U_4(3).2^2$, each with a fourgroup E for which $C_X(E)$, of order 40 and 80 respectively, has a normal subgroup of order five while $O(X) = 1$. Correction: $\mathcal{C}_0$ is enlarged by exactly these two groups, both of odd Lie type.
 - Part I, Proposition 1.1 and (1.2); the same identity recurs in Gilman–Solomon. Case 36. The proposition asserts a 2-component L_2 of X with $L_1 = L_{2'}(C_{L_2 L_2^t}(t))$. In $X = Sp_4(q)$, $q \geq 5$ odd, with t having two nondegenerate symplectic eigenplanes, $C_X(t) \cong SL_2(q) \times SL_2(q)$ and either factor can be L_1 , whereas the asserted layer is the product of both. Correction: the equality is replaced by the assertion that L_1 is a 2-component of $C_{L_2 L_2^t}(t)$, with whole-layer equality retained in the interchanged-component case.

Intermediate steps and additional arguments. Other entries give local configurations that invalidate an intermediate inference based on the information used at that point. Such a configuration may fail additional hypotheses of the surrounding theorem and therefore need not contradict that theorem. Examples include Cases 1, 8, 12, 14, 16, 19, 21, 22, 28, 31, 37, 49, 73, 76, 77, 82, 88 and 94. Further entries concern exceptional cases, hypotheses needed in an application, or arguments not supplied in the cited passage.

Role in the classification. CFSG is correct, and the formalization establishes the same classification. For each departure listed here, AxiomProver supplied a corrected statement, a supplementary argument, or an alternative route sufficient for the application described. In several cases the result obtained is weaker or more specific than the printed assertion, as in Cases 2, 17, 25, 39, 40 and 71. The scope of each resolution is part of its mathematical content. These entries do not assert that every stronger statement in the cited works has been recovered in its original generality.

3 Index to the cases

The cases are grouped by the extent of the change to the argument. Page numbers link to the detailed entries.

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4 Part A: Structural changes and alternative proofs

These entries concern changes to a major reduction, replacements of a chain of arguments, or alternative routes to the result needed in an application.

Case 1. A closure assertion and an alternative route to the Unbalanced Group Theorem

Literature. Part II, the closure assertion after (5.2), p. 136; its use in Proposition 5.9(ii), pp. 145–148. [47, 19].

Point of departure. Walter takes $\mathcal{L}_0$ to consist of the groups $\mathrm{PSL}_2(q)$ with q odd, excluding $q = 9$ and Fermat or Mersenne primes. The passage following (5.2) asserts that a nontrivial 2-layer obtained by centralizing an involution remains of this type. Let $S = \mathrm{PSL}_2(r^2)$ and let σ be the field involution. Then

$$C_S(\sigma) \cong \mathrm{PGL}_2(r), \quad E(C_S(\sigma)) \cong \mathrm{PSL}_2(r).$$

The actor is realized in $S \rtimes \langle \sigma \rangle$; it is not an inner involution of S . For $r = 7$, the source $\mathrm{PSL}_2(49)$ belongs to $\mathcal{L}_0$, whereas the component $\mathrm{PSL}_2(7)$ does not. The same problem occurs at other excluded subfield parameters. This disproves the intermediate closure assertion and leaves its use in Proposition 5.9(ii) unjustified; it is not a counterexample to UGT or to the proposition with all its ambient hypotheses.

Resolution. The replacement bypasses this closure argument using a comparison theorem for actual local 2-components. In a simple $\mathcal{K}$ -proper group, fix a simple top $T \cong A_7$ or $\mathrm{PSL}_2(q)$, q odd, and assume that the pointed-maximal family of top T has 2-rank two. The argument chooses an optimal pointed occurrence, proves isomorphic Gilman–Solomon transport, and extends it to marked fixed layers. Relative correspondence identities, absorption in proper subgroups, and a signalizer/embedding argument exclude a swapped marked pair. Thus the stronger comparison relation, including mirror steps, preserves the top. This proves the exclusion of tops required in Foote’s argument and hence the simple-group UGT reduction through Foote and the remaining recognition theorems. The replacement is described in [19], Theorem 2.3 and Section 9. It does not claim to reprove Walter’s unrestricted B-theorem by the printed argument.

Case 2. Abstract $\mathrm{GF}(q)$ -type theory admits semifield counterexamples

Literature. §1, Lemma 1.2, Corollary 1.3 and Theorem 1.6; their use in the Main Theorem [44].

Point of departure. The abstract theory infers field multiplication from the partition and centralizer properties of certain 2-groups. Those conditions allow non-Desarguesian spread data. An explicit order-4096 example is built on V^3 , $V = \mathbb{F}_2^4$, with multiplication

$$(u, w, z)(u', w', z') = (u + u', w + w', z + z' + \beta(u, w')),$$

where β is a nonsingular bilinear pairing from such a spread. Its two distinguished elementary abelian subgroups have order 256 and generate the group, but a mixed-element centralizer has the wrong structure for the asserted $\mathrm{GF}(16)$ model. Consequently the abstract lemma cannot supply the field structure used later.

Resolution. The replacement obtains field structures from the additional local hypotheses of the Main Theorem, using Timmesfeld’s earlier Math. Z. 161 (1978), (2.18). Opposite conjugates $U = V^g$ inside Q are partitioned by compatibility of their multiplication data. Each compatibility

class generates a Heisenberg group associated with a nondegenerate hyperbolic quadratic space over $\mathbb{F}_{2^n}$, hence a central product of Sylow 2-subgroups of $L_3(2^n)$ with common centre V . Different classes commute, so their central products assemble to give the required description of Q . This proves the application without reinstating the false abstract theory.

Case 3. Two natural-module regimes escape the FF reduction

Literature. Aschbacher–Smith, Vol. I, C.4.8(1)–(2); Vol. II, Theorem 4.2.13, pp. 611–612, and Lemma 4.4.6(1), p. 620 [16, 17].

Point of departure. The FF-module reduction misses two different natural-module regimes. In the two-line alternative, a section can be $N \oplus N$, with N natural for $\mathrm{SL}_2(q)$, $q > 2$. A nontrivial elementary abelian actor has order at most q , but its fixed-space index on this section is q^2 , so the required FF inequality fails. In the other regime the section is N^k , with N natural for $L_m(2)$ and $k \geq m$, notably $m = 3, 4$; the same FF conclusion is unavailable. Local examples do not satisfy all the global quasithin hypotheses, but show that local module analysis alone cannot exclude these cases. A related enlargement $I \leq I_1$ also loses information: the block property of $O^2(I_1 \cap L)$ need not descend to $O^2(I \cap L)$.

Resolution. The replacement retains only those classification conclusions that survive passage to a maximal overgroup and treats the two regimes separately. For two natural summands, first-cohomology vanishing separates the natural and fixed central parts of $\Omega_1 Z(R)$; the ambient restriction on 2-local overgroups and the quasithin rank bound exclude the surviving central kernel. For large multiplicity, the remaining tensor cases are reduced to $(n, m) = (4, 3), (5, 3), (5, 4)$. A coset-graph argument shows that a critical pair would force a special linear action on the wrong multiplicity space. Without a critical pair, a nontrivial normal 2-subgroup contradicts the global overgroup restriction. The exceptional $(5, 3)$ case is supplied separately. These independent repair branches belong to one defective FF reduction; they are counted here as one case, and no arbitrary subgroup-descent property for blocks is asserted. The unsupported block conclusions for the original smaller subgroup are not restored.

Case 4. A semilinear case treated as though it were linear

Literature. (3.4), core step (1'), printed pp. 868–869 [31].

Point of departure. The proof of (3.4) uses invariance of a subgroup of order q^2 in a natural $\mathrm{SL}_3(q)$ -module to infer that it contains the required fixed line. Such an inference needs linearity over $\mathbb{F}_q$, or a sufficiently large linear unipotent action. The case being considered permits field automorphisms, so neither condition has been established. Indeed, the linear case already gives the desired core inclusion; the unresolved part of the printed argument is precisely the semilinear case. This is a gap in the reduction, not a counterexample to the main pushing-up theorem.

Resolution. The replacement avoids unrestricted (3.4). It reorganizes the reduction by induction on the Sylow subgroup and proves the necessary semilinear exclusions directly. In the symplectic case the replacement separates the two critical-pair shapes: quadraticity forces the moved space into the relevant fixed line in one shape, while root-group coordinates give the corresponding containment in the other. Both yield $[Z_\mu^*, Z_\alpha] \leq Z_\mu$. A Burnside-type argument then excludes the nonlinear action. The unique-centralizer recognition route consumes these narrower exclusions; it does not assume the defective core assertion. The unrestricted printed core lemma is not asserted to have been repaired.

Case 5. Centrality in the minimal-parabolic theorem and its later application

Literature. Aschbacher–Smith, Vol. I, Theorem C.1.34(4), pp. 136–137, and Vol. II, Lemma 15.3.23, p. 1128 [16, 17].

Point of departure. The asserted containment $C_Q(L) \leq Z(G_0)$ in C.1.34(4) is false in a verified $F_4(2)$ parabolic configuration. The fixed part of the unipotent module need not be centralized by the whole group used in the theorem. A long-root subgroup lies in $C_Q(L)$ but has a nontrivial commutator with another root subgroup in the Sylow 2-subgroup. The underlying Meierfrankenfeld–Stellmacher result gives centrality only relative to a smaller subgroup. In the later use at 15.3.23, the claimed noncontainment of a commutator in $C_Q(L)$ likewise has no purely local proof; the exceptional configuration exposes the missing distinction.

Resolution. The corrected version of C.1.34 deletes the false centrality assertion and retains the conclusions supported by the minimal-parabolic analysis. The quasithin arguments use this weaker theorem. In the problematic Chapter 15 branch, the proof applies the corrected minimal-parabolic result to the larger, globally controlled pair (LS, S) rather than to the insufficient pair (LR, R) . The ambient quasithin hypotheses and maximal-overgroup information then supply the required noncentral commutator and close the branch. This is an additional global exclusion, not a proof of the false local centrality statement.

Case 6. A field-size parameter is not preserved through nonsplit 2-local covers

Literature. Aschbacher–Smith, Vol. I, Lemma E.2.2, pp. 215–216; F.7.13(2), pp. 314–315; Vol. II, Lemmas 6.1.5, 6.1.19, 6.1.25 and 13.2.8 [16, 17].

Point of departure. The equality $n(H) = n$ asserted in E.2.2 fails for nonsplit configurations such as $\mathrm{SL}_2(5)$ and $2^4 : \mathrm{SL}_2(5)$ with the natural $L_2(4)$ action. The local parameter can be one while the field parameter is two. Several later arguments infer a lower bound for the local parameter from the field size and thereby omit precisely these nonsplit cases. The failures in F.7.13 and the Chapter 6 lemmas are consequences of this common incorrect identification, rather than independent failures of the underlying classification.

Resolution. The corrected statement distinguishes the two quantities, retaining the valid bound $n(H) \leq n$ and the quotient-level equality where it holds. The case division in Chapters 5–6 is reorganized by the actual field size of the Lie component instead of the invalid equality. Applications that require a lower bound prove it from their additional hypotheses. In 13.2.8, for example, the specific centralizer condition identifies the relevant 2-core, and freeness of the resulting fourgroup module supplies the local splitting needed for the rank estimate. The corrected F.7.13 has an additional hypothesis, which is established in its applications. No claim is made that arbitrary nonsplit covers satisfy the original equality.

Case 7. Split and nonsplit $L_2(4)$ -blocks

Literature. Lemma 2.16(2), (5) and (8), especially the nonsplit case on pp. 507–508 [40].

Point of departure. In the exceptional $n = 2$ case the proof claims that W is elementary abelian and splits as $V \oplus Z$ as a module. There are nonsplit blocks with $X/V \cong SL_2(5)$ for which W/V is trivial as a module but has no invariant complement. Examples of orders 1920 and 3840 respectively have $W \cong C_2^5$ and $W \cong C_4 \times C_2^4$. The associated Sylow 2-group does not have the quaternion-product quotient used in the proof. Consequently parts (5) and (8), not just a sentence in the proof of (2), fail in this nonsplit situation.

Resolution. The replacement separates the split case $W \leq VC_T(X)$ from the nonsplit case, rather than treating the latter as class two. In the nonsplit application, it analyzes involutions and maximal elementary abelian subgroups of the actual Sylow 2-group, proves rigidity of the relevant families, and establishes weak closure of the specified subgroup E . Timmesfeld’s classification and the available rank and order bounds then exclude that nonsplit configuration under the stronger ambient application hypotheses. The false assertions are not restored for arbitrary blocks: the explicit nonsplit examples remain valid. Together with the corrected statement concerning W , this exclusion gives the required pushing-up conclusion.

Case 8. A quotient-rank estimate does not give the required involution rank

Literature. Theorem 10.4, especially (10.14) and the final paragraph, pp. 598–601 [8].

Point of departure. The final paragraph compares the dimension of a quotient by an involution commutator space with the minimum involution rank. These are different quantities. In the order-three branch the preceding reduction uses an element of order seven, giving only a three-dimensional bound where the contradiction requires a bound of two. Moreover, passing from $V/C_V(G)$ back to V can add the dimension of $[V, t] \cap C_V(G)$. The four-dimensional affine-semilinear example demonstrates the failure of the printed intermediate inequality, but violates the stronger hypothesis of the theorem and is not a theorem counterexample.

Resolution. The replacement forms, for each member X of the given family, the subgroup generated by X and all family members that fail to commute with it. A pair theorem and a triple theorem show that this subgroup is either X itself or one Frobenius group of order 21, and that noncommuting members determine the same subgroup. Distinct resulting factors commute. Their moved spaces lie in each other’s fixed spaces, which proves independence of both the group factors and the module summands. This proves the full decomposition directly, bypassing the disputed quotient-rank argument.

Case 9. The $3C$ and $4B$ cases in the Monster graph argument

Literature. Aschbacher–Segev, Extending morphisms of groups and graphs, (8.13)(2) and (8.15), Case 2, pp. 321–322 [13].

Point of departure. For a pair whose product has Monster class $3C$, the common centralizer involves the Thompson group and need not contain the required $2A$ involution. Hence the common-neighbour assertion in (8.13)(2) is false in this case. The corresponding $2A$ -pure S_4 configuration exists, so the branch cannot simply be excluded. In (8.15), the $4B$ argument also requires exact commutation although only commutation modulo a central involution has been obtained. These defects leave a gap in the claimed homotopy reduction of short cycles in the Monster graph.

Resolution. The replacement proves a more appropriate statement: any two length-three paths between the relevant $3C$ pair are homotopic. It uses generation of the Thompson centralizer by its involution centralizers and the actual S_4 configuration through the pair. The four-cycle argument is then reorganized according to whether a close pair or a suitable centralizing vertex is available; otherwise the new S_4 length-three-path reduction supplies the homotopy. This yields the required triangulability without demanding a nonexistent common $2A$ neighbour or upgrading commutation modulo a centre to exact commutation. These replace, rather than prove, the false common-neighbour assertion.

Case 10. Two $A_5(2)$ subgroups can generate $E_6(2)$

Literature. Lemma 5.11, proof of the $A_5(2)$ half-end-node subcase, printed pp. 481–482; comparison with Lemma 5.10 [32].

Point of departure. Inside $E_6(2)$, two root-centralizer components $K_1, K_2 \cong SL_6(2)$ can intersect in the prescribed $SL_4(2)$ component and have joined reflection action $W(B_4)$, while $\langle K_1, K_2 \rangle = E_6(2)$. The printed half-end-node list does not allow this outcome. The proposed reduction to an exercise in the classical A -type model therefore cannot establish the claimed conclusion. Prime divisors also separate the actual exceptional join from the indicated larger linear groups. This is a missing recognition branch; it does not refute the main theorem that the resulting group is of Lie type.

Resolution. The corrected argument replaces the universal half-end-node assertion by a construction that retains the exceptional case. Actual $E_6(2)$ gluing data supply the omitted completion, and the subsequent proof treats its centralizers and normalizers separately. The classical argument is used only with its additional automizer-containment condition, so it cannot discard an ambient $W(F_4)$ action merely because the two selected reflection actions generate $W(B_4)$. The subsequent argument uses a dedicated half- E_6 construction, verified by finite computation, instead of the false exhaustive list.

Case 11. Shear automorphisms and transvection classes

Literature. Aschbacher–Seitz, Involutions in Chevalley groups, (11.13)–(11.14), symplectic case; and On groups with a standard component of known type, (7.5), p. 455 [14, 15].

Point of departure. For a symplectic group the unipotent group T in the relevant centralizer is abelian. A nontrivial cohomology class in $H^1(L, T)$ therefore gives a shear automorphism of $T \rtimes L$: it fixes T and changes the complement by a cocycle. In $Sp_4(4)$ such automorphisms send a transvection in the complement to an involution of rank two. Consequently the assertion that arbitrary automorphisms of this centralizer preserve the required transvection class is false. The printed matrix calculation implicitly preserves a zero block that a shear need not preserve.

Resolution. The replacement chooses an odd-order element y in the actual intersection $A \cap A^g$ and takes $Y = \langle y \rangle$ instead of the entire earlier subgroup W . The semisimple centralizer in this group has $O_2(C_{A^g}(y)) = 1$, which proves that $C_G(YT)$ cannot normalize the designated 2-subgroup R . The choice is made so that $E(C_A(Y))$ remains the required quasisimple Levi component and no involutory automorphism centralizes the needed product. The valid core of (7.5) now applies; the $Sp_4(4)$ case receives its own explicit choice. This avoids the false involution-centralizer rigidity. This also bypasses the earlier transvection-rigidity assertion; the two historical occurrences of the shear defect are counted once. It does not prove unrestricted automorphism rigidity.

Case 12. A signalizer that cannot vanish in the intended local model

Literature. Lemma 6.3 and Theorem 6.4, the $n=11$ signalizer argument, pp. 518–520 [36].

Point of departure. The printed completion W contains $O(C_G(Z))$, rather than only the odd cores of individual involution centralizers. In the concrete local model $G_0 = \text{Ly} \times C_2$, the relevant subgroup Z has an eight-point support in the $\hat{A}_{11}$ involution centralizer. The three remaining points supply a nontrivial 3-cycle in $O(C_{G_0}(Z))$, although the relevant individual involution centralizers have trivial odd core. Thus $W \neq 1$ in this model, defeating the local inference used in Lemma 6.3 and Theorem 6.4. The model is not a simple minimal counterexample; it does not refute the classification theorem under all its hypotheses.

Resolution. The replacement uses the subgroup generated by the individual odd cores $O(C_G(b))$ for the relevant involutions b , rather than the larger printed completion. Its definition therefore avoids the extraneous three-point contribution. Fusion analysis, together with the appropriate Aschbacher and Bender theorems, proves that this modified signalizer is trivial. The argument then obtains the needed odd-core vanishing and continues through the recognition conclusion. The resulting case theorem gives the required conclusion directly, without assuming the original equality $W = 1$.

Case 13. Closure of one conjugacy class is not strong closure

Literature. (5.3) and (5.7), the 2B and 2D centralizer arguments [41].

Point of departure. The arguments establish fusion control for one class of involutions and then apply a theorem requiring strong closure of the whole subgroup. The subgroups in question have other involution classes, and the extraspecial subgroup in (5.3) also contains elements of order four. Generation by the controlled class does not fill the gap: a product can fuse into a Sylow subgroup even when its factors do not. The elementary example S_4 , with a four-group generated by two disjoint transpositions, already separates these notions. The missing condition matters to the subsequent centralizer and order computations.

Resolution. The replacement proves full strong closure first. A general lemma combines a block property for the distinguished involutions with a fixed-point condition on the relevant module. It controls the other classes rather than extrapolating from the generating class. In the extraspecial case this supplies the required statement in the quotient by the central involution, where Goldschmidt's abelian strongly-closed theorem can be applied. Separate arguments give the 2B and 2D centralizer containments needed for the recognition theorem. The added argument proves the required strong-closure property.

Case 14. The rank-three transfer step requires a different global exclusion

Literature. (11.6), second case, p. 94 [11].

Point of departure. The proof controls conjugacy of $\langle a \rangle$ inside H and then invokes transfer as though the whole relevant subgroup W acted trivially on A/B . The printed information controls only $H \cap W$; the remaining elements may invert the quotient. A local model with $W^* \cong E_9 : D_8$ exhibits the obstruction, and a larger model based on two $L_2(64)$ factors shows that the displayed local assertions do not themselves exclude it. These examples do not satisfy all the global hypotheses of the rank-three classification, so they refute the transfer inference rather than its final classification conclusion.

Resolution. The replacement argument proves the residual case impossible under the extra condition used by the classification application, namely $A \notin \mathcal{T}$ or $A \in \mathcal{S}$. This condition is supplied on that route by the earlier reduction, rather than inserted as a universal amendment to (9.1). In the difficult residue, critical-subgroup and fixed-space arguments force nontrivial O_2 in a relevant local core. Applying 2-balance at the specified three points, with the nontrivial commutator action, forces $A \in \mathcal{T}$, contradicting the branch assumption. The $\mathcal{S}$ branch is treated separately. Thus the required restricted conclusion is proved without repairing the invalid transfer calculation.

Case 15. A critical-subgroup argument replaces the final $2.A_7$ commutation step

Literature. (13.5), final step in the $2.A_7$ case, p. 107 [11].

Point of departure. The proof moves from a centralizing relation for an elementary abelian actor to a claim that an element c centralizes the relevant component. This omits the possibility that c permutes components. The last asserted contradiction also uses a centralizer relation not established for the chosen element. Passing to a quotient by the odd core does not automatically fix either problem: that core can have nontrivial 3-part, so a 3-element cannot simply be lifted as though the kernel had order prime to three.

Resolution. The replacement works inside $C_G(c)$ with the actual odd core retained. The list of known groups restricts the component types to He , $O'N$ and J_4 . A c -invariant component, or the fixed diagonal of a three-cycle on components, is lifted through an invariant Sylow 3-subgroup. A critical subgroup of its intersection with the odd core supplies a module on which the available local 3-rank bounds limit involution fixed spaces. The elementary abelian 2-subgroups in He , J_4 , and the $4^3 : 7$ structure in $O'N$, violate those bounds unless the perfect lift centralizes the Sylow subgroup and c . The remaining odd-prime action then gives the desired contradiction.

Case 16. The Held diagonal does not establish ordinary component maximality

Literature. Lemma 3.5, the swapped-component/diagonal case, pp. 112–113 [34].

Point of departure. In the diagonal case, an involution interchanges two components and the proof passes from the fixed diagonal to assertions that one ambient factor is maximal and has centralizer of 2-rank greater than one. Neither assertion follows from the displayed diagonal configuration. A wreath product $He \wr C_2$ illustrates why the local configuration alone does not justify that passage. It is a diagnostic local model, not a counterexample to the theorem under its full simple-group hypotheses.

Resolution. The replacement keeps the actual two components and their involution action, and applies the component theorem and classical involution theorem in the ambient simple-group setting. The Held specialization of the component-theoretic alternatives, together with the relevant local conclusion corresponding to Lemma 3.15(b), excludes the swapped configuration without declaring the fixed diagonal maximal. The result proved is the Held-specific swapped-component statement, subsequently used by the Held maximality argument. The results on known groups and the classical involution theorem are used explicitly, and an abstract assertion that every diagonal component is maximal is neither proved nor needed.

Case 17. A fixed-component intersection in place of global normality

Literature. Corollary 2.14(1) and (3), pp. 104–105 [34].

Point of departure. Corollary 2.14(1) asserts normality of the designated component in the ambient group. Let Q be a central 2^2 -cover of $L_3(4)$ and consider $Q \wr C_2$. The factor $Q \times 1$ occurs in the appropriate involution centralizer, while the swapping involution moves it. The group has trivial odd core, and the proposed Held outcome cannot occur by order considerations. The proof's restriction to a smaller generated group does not preserve the needed minimality. The subsequent part (3) also uses an identification chain which does not, by itself, preserve actual component position.

Resolution. The replacement proves only the intersection statement needed in the pump-up argument. For a quasisimple known group Q with 2-group centre and an involution acting on it, consider a proper fixed 2-component P of top $L_3(4)$ whose relevant central subgroup has 2-rank at least two. The required conclusion is $P \cap Z(Q) = 1$. The catalogue of proper fixed $L_3(4)$ components reduces this to $L_3(16)$, Suz , $O'N$ and He . Their multiplier and fixed-component structures eliminate a rank-two central intersection. This removes the false global normality statement and does not claim a general Held identification from the original hypotheses.

Case 18. A proper centralizer layer replaces the unsupported generated overgroup

Literature. (26.7) and (28.2)–(28.3), the triangle and Case N arguments, pp. 450, 453–454 [4].

Point of departure. The final link analysis forms $X = \langle \Omega(K) \rangle$ and needs both $X \neq G$ and $X = O(X)\langle \Sigma \rangle$. Neither is established in the relevant cases. In triangle configurations, the component graph may have no relevant edges and Σ may have only one member; in Case N, the map from the original family to quotient components can have nontrivial fibres. Central products built from $SL_2(3)$ show why injectivity cannot be inferred from the abstract labels. Consequently neither the induction nor the subsequent orbit argument can simply be applied to X .

Resolution. The replacement instead forms the neighbourhood layer

$$Y = \langle C_N(z) : N \in \Omega(K) \rangle \leq C_G(z).$$

This group is proper for the actual involution z , so the induction hypothesis is available without first proving $\langle \Omega(K) \rangle \neq G$. The analysis retains the actual family and its fibres and treats its possible thin and S_3 layers separately. In the thin case, Part I (7.10) restricts the relevant permutation image to the fourgroup or S_4 configurations, which are excluded using Theorem F and Part II (26.2); the S_3 case has its own local analysis. This avoids both false generation and unjustified injectivity.

Case 19. The semidihedral link needs a frame and fusion argument

Literature. (26.10), the $L_2(q^2)$ link, Case A and the residual inner/outer transfer cases, pp. 450–451 [4].

Point of departure. For $q \geq 5$, the second alternative of Case A assumes that a suitable outer involution centralizes the required quaternion subgroups and satisfies the signalizer hypotheses. Field and diagonal automorphisms need not do this; the $q^2 = 25$ model already defeats the unrestricted assertion. After this alternative is removed, the printed transfer discussion still lacks the required control of outside involutions and their fusion.

Resolution. The replacement organizes the actual commuting involutions into a frame and follows their action on the two quaternion links. It proves the pair property, faithfulness of the action on the links, and the required normality modulo odd cores in the corresponding centralizers. Explicit automorphism-group and finite-model calculations then provide the weak-closure and fusion statements that transfer needs, including residual cases where an involution-free coset requires higher-order transfer information. The unsupported alternative in Case A is treated by this analysis of the remaining cases. The required link theorem follows from these lemmas concerning the frame and the odd cores, together with its stated induction hypotheses. The finite-model calculations remain separate ingredients of the proof.

Case 20. A rotation normalizes the two factors instead of swapping them

Literature. Lemmas 5.16(2), 5.18 and their use in 5.19–5.20 [9].

Point of departure. In the natural $PSO_8^+(3)$ model, an order-four rotation of an anisotropic plane centralizes the designated $\Omega_6^-(3)$ component. On the complementary plus-type four-space it has determinant one, so it preserves the two $SL_2(3)$ factors; it does not interchange them as Lemma 5.16(2) claims. The proof of 5.18 invokes this swap rule repeatedly. A related assertion in 5.18(1) also fails in the inner-diagonal local model. These nonsimple models refute the claimed local deductions, not the final theorem under its full simple-group hypotheses.

Resolution. The replacement discards the swap clause and does not use 5.18(1). It proves the needed rank bound 5.18(2) from the valid parts of 5.14–5.17, a parity argument and explicit finite centralizer and normalizer facts. It then proves 5.19 by a Goldschmidt–Yoshida counting construction, including a support-two calculation in the appropriate permutation action, instead of deriving it from 5.18(1). The final application of Goldschmidt’s theorem gives 5.20. Thus the recognition conclusion follows from a replacement argument that retains actual actions on the factors and does not silently restore the false swap.

Case 21. The seven-point centralizer argument needs a different route

Literature. Lemma 5.10, seven-point case, printed p. 417 [9].

Point of departure. Several steps in the seven-point proof are not established by the preceding local facts. The printed fusion argument checks involutions but asserts strong closure in a subgroup that may contain elements of order four. More decisively, an odd-order automorphism centralizing a Sylow four-group need not act trivially modulo the odd core: the field automorphism of order three of $PSL_2(27)$ centralizes the subfield $PSL_2(3)$ and hence a Sylow four-group. Consequently the final inference to $[K_i, Y] \leq O(Y)$ is invalid without additional information about the actual configuration.

Resolution. The replacement proves the involution fusion restriction by spinor norms and component balance, handles order-four elements by a determinant calculation over $\mathbb{F}_3$, and chooses component lifts inside the appropriate centralizer. It then uses the classified dihedral or abelian Sylow shapes of the remaining quotient. A new local lemma shows that elements of order prime to six in the relevant common centralizer centralize the original component; order arguments exclude the unwanted field actions. This proves the final commutator bound and both residual cases of Lemma 5.10 without the printed transfer and tight-embedding shortcuts.

Case 22. The Hessian contradiction cannot be reduced to the asserted centralizer

Literature. (14.6), within the Hessian configuration (14.4)–(14.8) [6].

Point of departure. The proof asserts a very small centralizer of an elementary abelian group of order nine inside $\langle y, u, z \rangle$. The available calculation controls only $\langle u, z \rangle$: the extra generator y need not normalize the original component. A local matrix model in $SL_9(2)$ has centralizer of order 81 instead of the asserted subgroup. That model does not satisfy every global hypothesis, so it is not a counterexample to the theorem. It does show that the printed computation cannot establish the needed exclusion from the local information it uses.

Resolution. The replacement excludes the Hessian root-group configuration itself. Its root groups carry two classes: groups of opposite classes at different nodes commute, whereas the commutator of same-class groups is a nontrivial root group of the opposite class at the third node. Choose three suitable noncollinear nodes. Two root groups commute with a third group, so their commutator must commute with it as well; the same-class rule simultaneously says that this commutator cannot centralize the third group. The two-step contradiction supplies the missing conclusion without requiring the unsupported small-centralizer assertion.

Case 23. Two torus actions require a comparison of kernels

Literature. (13.16) and the row-7 conclusion of (13.22) [6].

Point of departure. The proof attempts to realize the action group $C_2 \times L_2(7)$ as a section of $GL_3(3)$ by letting two local Weyl groups act on a common elementary abelian 3-group. Equality of the action kernels on each generator subgroup does not establish equality, or either required inclusion, for the group they generate. The kernel on the original p -torsion can act nontrivially on the 3-torsion. Thus the final order contradiction involving $7 \nmid |GL_3(3)|$ has no justified action quotient to which it can be applied.

Resolution. The replacement bypasses (13.15)–(13.19) and excludes the relevant $C_2 \times GL_3(2)$ datum directly in the symplectic row. It first calibrates the actual root frame. For odd field exponent, the pair theorem supplies common components for the required reflections, and the resulting line-commutation relations contradict the datum. For even field exponent, the order-three subgroups of the component tori provide a different contradiction. The resulting theorem resolves the row-7 case of (13.22), and no comparison of the two unproved global kernels is inserted into the argument. The standalone assertion (13.16) is bypassed, not repaired as an independent lemma.

Case 24. Additional linear-group configurations in HSS (3.5)

Literature. Huang–Stellmacher–Stroth, Proposition (3.5), its claims (2), (5), and final linear-group steps, pp. 94–97 [28].

Point of departure. The proof of (3.5) reduces to small binary linear groups but does not justify the exclusions used at the end. For critical distance three, the stated orthogonal-module data leave a consistent group of order sixty-four with an $L_3(2)$ or Heawood-type configuration. For larger critical distance, the commutator images need not have the mutual commutation asserted in the proof, and one case is omitted. The earlier claim (5) also needs a corrected formulation. Thus an appeal to the structure of $GL_4(2)$ does not close either range with the hypotheses actually established.

Resolution. The repair separates the two corrected configurations in claim (5) and proves the needed plane-preservation statement at the transported vertex in each one. This supplies the revised form of the later claim before either case is excluded. At distance three, a separate finite linear-group argument treats the actual order-sixty-four subgroup in a five-dimensional space; it does not assume that the inconsistent conclusion already follows in four dimensions. Together, these arguments exclude the orthogonal case. The stronger local calculation is available even where the classification argument uses only the weaker dichotomy.

Case 25. An alternative treatment of the perfect local cases

Literature. Aschbacher–Smith, Vol. II, Lemma 15.3.68(2), pp. 1154–1155, and the closing argument for Theorem 15.3.1 [16, 17].

Point of departure. The proof of 15.3.68(2) uses the equality $Q_1 \cap Q_2 = R$, which fails in the relevant local configuration. The available inclusion of Q_0 in the intersection consequently does not imply that Q_0 centralizes V . A local model $2^6 : (L_3(2) \times 2)$ in a parabolic of $L_5(2)$ has the required natural-plus-dual module and violates the asserted conclusion. The end of Theorem 15.3.1 relies on part (2) to exclude the surviving perfect local cases, so those cases remain unexcluded by the printed argument.

Resolution. The corrected version of 15.3.68 discards the unsupported part (2). The remaining $L_3(2)$ configuration is handled globally by applying 15.3.50 to the actual maximal subgroup M_c and the corrected C.1.34 to the appropriate minimal-parabolic pair. The resulting normalizer of $V_E V_E^s$ contains $\langle T, X_E, Y_F \rangle$, contradicting the established description $\mathcal{M}(T) = \{M, M_c\}$. The replacement proof thus uses the ambient maximal-overgroup information that is absent from the counterexample’s local data. It does not attempt to prove the false intersection equality or to exclude the perfect configurations using 15.3.68(2). The discarded local assertion is not restored.

Case 26. A relative ZJ theorem without the required Frattini condition

Literature. Chapter II, Section 7, Theorem 2 (relative ZJ theorem), pp. 61–62 [1].

Point of departure. The relative theorem extends a Sylow-subgroup ZJ argument to a general subgroup R . The proof uses Frattini factorizations involving $R \cap L$ for normal L , although the stated conditions do not make this intersection a Sylow subgroup of the normal subgroup L . A wreath-product counterexample takes $G = H \wr C_2$ and $R = S \times O_p(H)$, where S is Sylow in a suitable p -stable constrained group H . If $Z(J(S)) \neq Z(J(O_p(H)))$, the factor-swapping element cannot normalize $Z(J(R))$. The claimed factorization fails even though the printed conditions on R hold.

Resolution. The replacement relative theorem makes the missing Frattini alternative explicit for the normal subgroups encountered by the induction. Under that condition, the original ZJ mechanism yields the required normalizer factorization. This is a corrected relative theorem; the false unrestricted statement is not used. In the documented height-one application, a structural description of the linear image proves the extra condition, so the corrected theorem can actually be applied. This proves the corrected theorem and the specified application. Other historical uses of the unrestricted relative result require separate verification of the added hypothesis.

Case 27. A cyclic-four exceptional A_7 configuration is not a fourgroup case

Literature. Chapter 9, Proposition 6.15 and the cyclic case of Lemma 8.13, pp. 418, 437–438 [25].

Point of departure. The exceptional A_7 branch has $Q \cong C_4$, but the proof invokes conclusions established for $Q \cong C_2^2$. A related passage treats K as quasisimple although only its quotient by the odd core is known to have top A_7 . Neither substitution is harmless: cyclic and elementary abelian order-four actors have different fixed-subgroup geometry, and an odd-core extension does not inherit every quasisimple centralizer lemma. This is a gap in that exceptional branch, not evidence against the general A_7 recognition statement.

Resolution. The replacement analyzes the actual exceptional triple directly. In the Sylow configuration $T = P \times P^a$ with $P \cong D_8$, the prescribed cyclic Q is not contained in P^a . One finds involutions in P^a whose action on the A_7 quotient has the specified transposition form. A suitable product e induces a transposition and is conjugate to a Sylow-central involution, forcing the Sylow centre of $C_G(e)$ to be $\langle e \rangle$. Known component and multiplier possibilities then contradict this centralizer structure. The resulting exceptional-case exclusion replaces the inapplicable fourgroup lemma and keeps the actual cover and odd core.

Case 28. An alternative involution-geometry argument for the Tits group

Literature. Aschbacher, The Tits group as a standard subgroup, Theorem (4.9), final paragraph, p. 240 [10].

Point of departure. The proof uses generation of $U_3(3)$ by three involutions to obtain a small commutator-space bound. That generation assertion is false, so the bound does not follow. Moreover the seven-dimensional $\mathbb{F}_5 U_3(3)$ modules excluded by the following paragraph do exist. Thus the remaining branch cannot be eliminated by the stated module-size argument. This is the final obstruction in Theorem (4.9), so an independent exclusion of the entire branch is required rather than a change to one numerical inequality.

Resolution. The replacement studies the involution geometry forced by the failure branch. From the actual involution centralizer and its specified fusion, it constructs three vertices P, Q, R and an element interchanging P and Q while fixing R . The local triangle analysis supplies an obstruction to precisely that permutation. The finite Tits-group facts used to construct and analyze the triangle are provided separately and composed with the global argument. This proves the required failure exclusion without asserting that $U_3(3)$ is three-involution generated or that its seven-dimensional modules do not exist.

Case 29. Nonsingular lifts are not covered by the involution-class argument

Literature. Timmesfeld, *Finite simple groups in which the generalized Fitting group of the centralizer of some involution is extraspecial*, (12.9), pp. 358–359, and its use in (12.13) [43].

Point of departure. The argument invokes involution results for elements whose squares have not been shown to be trivial. If a nonsingular vector occurs in the relevant fixed space, its lift satisfies $x^2 = z$ and the induced involution has class c_2 , not the asserted class a_2 . A direct commutator calculation distinguishes these classes. The paper gives no argument that the entire fixed space is totally singular, so the all- a_2 fourgroup used in the following reduction is not established. This is a conditional obstruction to the printed inference, not an exhibited finite group satisfying the whole Section 12 configuration and refuting its conclusion.

Resolution. The replacement avoids (12.9) altogether in the affected branch. It constructs a generating configuration consisting of three and two suitable subgroups and estimates the dimension moved by their joint action. The resulting bound $\dim[\tilde{Q}, \bar{X}] \leq 7$ is strictly less than the ambient dimension $2n$, yielding the reducibility needed by the later argument. The remaining small unitary width-four case is treated explicitly. No assumption that a nonsingular vector induces class a_2 is used in this replacement.

Case 30. Width-dependent fixed-space and fusion arguments

Literature. Correction of (3.11)(1), including its final fusion argument, pp. 413–414 [45].

Point of departure. The correction seeks a fixed-space equality $C_{\tilde{Q}}(\bar{L}) = \langle \tilde{a} \rangle$ uniformly in the width parameter. The statement is false at width three. At width six, the assertion that the designated element is the only element of its specified type is incompatible with the remaining order-four possibility for D . The final appeal to Shult’s fusion theorem does not eliminate that possibility as written. These are connected defects in one repaired proof, not three independent classification statements.

Resolution. The replacement separates the statements that hold in all widths from the stronger equality. For width at least three it proves weak closure and uniqueness of the relevant singular vector; the full fixed-space equality is asserted only in the range where the dimension argument proves it. A two-point analysis eliminates the order-two and order-four possibilities for D , while width five is excluded by comparing the global and local fixed-space bounds. The final fusion step is supplied through Timmesfeld’s weakly-closed transitive-involution theorem with its hypotheses verified. Thus the correction’s intended application survives, but its uniform small-width assertion is not retained.

Case 31. The orthogonal centralizer module is not natural

Literature. (2.10), Case III (width six), pp. 272–273 [35].

Point of departure. The proof derives a contradiction by declaring a four-dimensional quotient both nonnatural and natural for $\Omega_4^+(2)$. The latter assertion is false in the relevant genuine orthogonal models: one S_3 factor acts trivially, and the module is a sum of two planes. The calculation in the larger orthogonal parabolic only controls one projection of the acting subgroup; the other component changes its action on the tensor product. Since the argument uses no global exclusion hypothesis at that point, it would also exclude actual configurations that the theorem must allow.

Resolution. The replacement invokes the global hypothesis omitted by the local calculation. It selects the required element from that hypothesis, uses the earlier decomposition, and constructs the auxiliary θ data in a hermitian model of $\Gamma U_6(2)$. A contradiction is proved for this whole configuration, without asserting that the original four-dimensional module is natural. The resulting Case III theorem is used in the width-six recognition argument. The required finite-model facts are verified by direct computation. The repaired proof establishes the case; it is not evidence that the printed tensor-product computation was correct.

Case 32. The root-type residual and swap cases require a new Section 4 argument

Literature. Griess–Mason–Seitz, Bender groups as standard subgroups, (2.9)(4), pp. 184–185, and (4.2)–(4.3), pp. 192–194 [33].

Point of departure. The root-type alternative occurs in the earlier reduction (2.9)(4), but the argument in (4.3) does not treat the case in which the relevant components are interchanged. In that case the asserted containment of a normalizer in the normalizer of the chosen subgroup need not hold. The residual root-type assertion in (4.2) is also left without the needed proof. Together these gaps leave part of the $L_2(2^f)$ standard-component analysis unclosed; they cannot be removed by treating the two component positions as identical.

Resolution. The replacement reconstructs the relevant part of Section 4 for $f \geq 3$. The root residue is controlled by the actual Borel and torus action, including transitivity on the appropriate root data and the bound on centralizers of field automorphisms. The component-swap alternative is then treated separately by weak closure, the applicable Goldschmidt theorem, and the preceding local intersection result. The theorem for the remaining case combines these two proved branches instead of assuming a fusion or normalizer assertion that fails in the swapped position.

Case 33. The $L_3(4)$ standard-component route bypasses a field-automorphism gap

Literature. Lemma 5, first step; English translation p. 6 [29].

Point of departure. The first step of Lemma 5 bounds the centralizer of an order-three element in the relevant quotient by a divisor of six. A field automorphism can centralize an additional involution, so the bound does not follow in the allowed configuration. Selecting a different order-three element is not an immediate remedy: in the induced S_5 extension of the A_5 Levi quotient, every 3-cycle has a commuting transposition. The same preliminary reduction also presupposes outputs of Lemmas 5 and 9, creating a second dependence problem within this one argument.

Resolution. The replacement classification argument does not cite the defective argument as a recognition theorem. It replaces that invocation by Cheng–Held’s local analysis of the actual standard $L_3(4)$ component and an O’Nan Sylow-characterization route. The former supplies the even-Sylow local structure with the component position and balance hypotheses retained; the latter performs the remaining recognition from that structure. This is an alternative route, not a corrected proof of Makhnev’s small-centralizer assertion. Bypassing this lemma alone does not complete the entire odd-standard theorem; its other required arguments are separate matters.

Case 34. An alternative route around two constituent calculations

Literature. Walter, Finite groups with Abelian Sylow 2-subgroups of order 8, Lemmas 7.4 and 7.8, pp. 363–368 [46].

Point of departure. The character argument in Lemma 7.4 does not exclude a seven-constituent triangular configuration before asserting the required eight-constituent structure. Lemma 7.8 similarly uses a common-constituent conclusion without disposing of a surviving decomposition. These are gaps in the displayed constituent analysis; they do not by themselves refute the final characterization theorem. Supplying a general statement about character multiplicities would not suffice unless it excludes the specific configurations with the hypotheses of these lemmas.

Resolution. The replacement argument does not attempt to complete those two character calculations. It uses the preceding local conclusions, including Proposition 6.6 and Lemma 6.5, and an independent Thompson identification of the relevant Ree-group base case. The order and local structural information supplied at that stage then yields the required alternative without passing through Sections 7–8. This gives an alternative proof of the classification application. This is a bypass; no claim is made that the printed constituent arguments have been repaired in their original generality.

5 Part B: Additional arguments and local corrections

These entries concern additional arguments, exceptional cases, or different local constructions used to establish a theorem or its application.

Case 35. The $L_4(3)$ and $U_4(3)$ cases in Walter's Theorem I

Literature. Introduction, definition of $\mathcal{C}_0$ and Theorem I, p. 2; compare (0.11)–(0.12) and Part II, Proposition 2.11. [47].

Point of departure. Theorem I asserts that a finite group X with a unique 2-component and with $O(C_X(E)) \not\leq O(X)$ for some nontrivial elementary abelian 2-subgroup E has its component in the listed class $\mathcal{C}_0$. The definition excludes both $\mathrm{PSL}_4(3)$ and $\mathrm{PSU}_4(3)$ and does not restore them elsewhere. The counterexamples are particular almost-simple extensions $X = L_4(3).2$ and $X = U_4(3).2^2$. Each has a fourgroup E for which $C_X(E)$ has a normal subgroup of order five, while $O(X) = 1$; the centralizers have orders 40 and 80, respectively. Their unique components therefore satisfy the hypothesis but not the printed conclusion. The distinction between a simple quotient and its permitted central covers does not remove these examples.

Resolution. Enlarge the simple-type list by precisely $\mathrm{PSL}_4(3)$ and $\mathrm{PSU}_4(3)$ for this conclusion. This is carried out in the definition of the corrected $\mathcal{C}_0$ and in the known-group base case. It leaves the separate definition of $\mathrm{Chev}^*(3)$ unchanged, since changing that definition would alter other theorems unnecessarily. Isomorphism transport, membership in the list of known groups, and the known-group B-property are extended over the two new alternatives; both are already groups of odd Lie type. Walter's later list of locally elementary-unbalanced types itself contains these two groups, which supports the correction. This repairs the omission in the printed list and its uses; it is not evidence, by itself, that every other step of the memoir's general Theorem I has been reproved.

Case 36. A componentwise fixed-layer correspondence

Literature. Walter, Part I, Proposition 1.1 and (1.2), pp. 6–7; Gilman–Solomon, preliminary correspondence discussion, p. 56 [47, 23, 27].

Point of departure. Proposition 1.1 asserts that, for a 2-component L_1 of $C_X(t)$, one can choose a 2-component L_2 of X with $L_1 = L_2'(C_{L_2 L_2^t}(t))$. Take $X = \mathrm{Sp}_4(q)$ with q odd and at least five, and choose t whose two eigenspaces are nondegenerate symplectic planes. Then X is its only component and

$$C_X(t) \cong \mathrm{SL}_2(q) \times \mathrm{SL}_2(q).$$

Either factor can be L_1 , whereas the asserted layer on the right is the product of both factors. The equality confuses one component with all components. The failure is in the invariant-component case; the diagonal case of an involution interchanging two components has a different conclusion. The same identity appears in Gilman–Solomon after adding an external central involution as a pointer.

Resolution. Replace the equality in Walter's proposition by the assertion that L_1 is a 2-component of $C_{L_2 L_2^t}(t)$, in particular $L_1 \leq L_2 L_2^t$. In the correspondence application, apply this to the actual perfect fixed layer. Retain whole-layer equality in the interchanged-component case, where the fixed group is diagonal modulo the odd core. The replacement uses this containment-based Gilman–Solomon relation and Harris's component-lifting Lemma 2.19 [27]. It also retains the witnessing involutions in Foote chains. A separate argument proves the maximality needed

in the classification after establishing its isomorphic-correspondence and Sylow hypotheses. Thus neither a universal fixed-layer identity nor an automatic equivalence of pointed and unpointed maximality is assumed. This does not assert a universal equivalence between the maximality notions.

Case 37. The Sylow-transfer step needs an involution in the action kernel

Literature. Gilman–Solomon, Finite groups with small unbalancing 2-components, Lemma 3.5, proof, p. 70 [23].

Point of departure. In the Sylow-transfer paragraph, the chosen central involution must act trivially on the component modulo its odd core. Centrality in a Sylow subgroup alone does not imply this. For example, in $A_5 \times C_2$, a nonidentity element of the Sylow fourgroup of A_5 is central in the product Sylow subgroup but its centralizer cannot provide the required component. This tests the unrestricted local choice, not the full hypotheses of Lemma 3.5. Moreover, preservation of the types of the chain vertices is weaker than the isomorphism at every comparison step needed in the later marked-family construction. Changing the witnessing involutions therefore requires a separate argument.

Resolution. The replacement establishes this stronger transport statement. It chooses the involution in the intersection of the Sylow centre with the nontrivial normal action kernel. It constructs unique covering components for nontrivial subgroups of that kernel, proves compatibility for nested subgroups, and transports both the actor and its component to a Sylow-maximal position. A strengthened diamond argument retains the source involution’s action on the common perfect fixed layer; this rules out a swapped target and proves the actual edge is isomorphic. Induction then handles arbitrary changes of witnessing involutions along a chain. The proof does not infer isomorphism at every step merely from preservation of the vertex types. The criterion alone does not assert that an arbitrary family satisfies its hypotheses.

Case 38. Cyclic fixed-space bounds do not extend to arbitrary elementary abelian actors

Literature. Corollary 2.11(b),(c) and equation (2.1); author manuscript pp. 9–10 [26].

Point of departure. In the inspected author-manuscript version, let $n = \dim V$ and $z = \dim V^G$. The first symmetric-square bound $\dim(\mathrm{Sym}^2 V)^G \leq z(n+1)/2$ is false for an elementary abelian group of rank two. Over $\mathbb{F}_3$, take $G = C_3^2$ acting on the square of the augmentation ideal in $\mathbb{F}_3[x, y]/(x^3, y^3)$. Then $n = 6$, $z = 1$, and the symmetric-square fixed space has dimension four, exceeding $7/2$. The corresponding first exterior-square bound and the general codimension forms also fail. The argument for a single cyclic generator does not extend to their intersection of fixed spaces.

Resolution. The replacement exhibits the counterexamples and states only the bounds actually proved. For elementary abelian groups it retains the binomial estimate

$$\dim(\mathrm{Sym}^2 V)^G \leq \binom{n-z}{2} + \binom{z+1}{2}$$

in odd characteristic, and the corresponding valid exterior-square bound. The stronger first codimension terms are proved separately for cyclic actors, with that hypothesis explicit. Later results are not permitted to call the false general versions. This is a correction to the module inequalities themselves; it does not assert that every later classification application in the

published chapter uses the invalid term. The disputed formulas were checked in the author manuscript; the final published wording was not compared.

Case 39. The characteristic-two identity-involution geometry has alternating links

Literature. Devillers–Gramlich–Mühlherr, Section 4, Lemmas 4.2–4.3 and Theorem 4.6 in arXiv:0711.3468v3; published article pp. 684–700 [20].

Point of departure. In the inspected author version, allowing the field involution to be the identity in characteristic two makes the link identification false. Over $\mathbb{F}_8$, take the standard symmetric form on a three-dimensional space and the nonsingular vector $(1, 1, 1)$. Its perpendicular space is alternating. Consequently the corresponding nonsingular point lies in no nondegenerate plane and is isolated in the proposed geometry, although the numerical field bound in the theorem is satisfied. Thus the nonempty-link and sphericity assertions do not hold for the full range of forms stated in this branch; a size estimate alone cannot repair them.

Resolution. The classification route uses only the genuine Hermitian case over a quadratic field extension and makes the nontrivial involution an explicit hypothesis. In that setting the nonzero Hermitian forms used in the residues have anisotropic vectors, so the alternating-link counterexample is excluded for a mathematical reason rather than by a stronger size bound. The local residue and field-four calculations needed by the Phan-type application are supplied in this restricted setting. This establishes the actual recognition input without claiming a repair of the false symmetric characteristic-two theorem in its original scope. The disputed unrestricted wording was checked in the author version; the final journal text was not compared.

Case 40. Additional Sylow hypotheses in the weak-closure theorem

Literature. Theorem 9.3 and (9.13), the omitted degree-two case [8].

Point of departure. The proof invokes a doubly transitive classification without dealing with a set of conjugates of size two. In $(U \oplus U) \rtimes S_5$, where U is the four-dimensional even-weight binary module, the relevant rank-three TI-subgroups give exactly this missing case. The normalizer containment and generation alternatives in the printed Theorem 9.3 both fail; the same model refutes the stated version of (9.13). The omission is therefore not merely a gap in a valid classification argument. The Sylow subgroup used locally need not be Sylow in the whole group.

Resolution. The version actually used adds that T is Sylow in G and that $N_G(T) \leq N_G(V)$. A Frattini argument then excludes a second conjugate of V normal in T , ruling out the degree-two case. The replacement proves this strengthened theorem through the necessary odd-transposition classification and feeds it into the Uniqueness Case applications, where the added hypotheses are available. The false general theorem and its deficient Section 9 proof are bypassed. An alternative unrestricted statement would retain a third degree-two outcome; that is not the version used.

Case 41. A chief factor does not absorb its prescribed centralizer automatically

Literature. Theorem 3.6(b), and its use in Theorems 3.7 and 3.10 [30].

Point of departure. The hypothesis only gives $X_2 \cap D \leq Y$, but conclusion (b) asserts equality and identifies the original pair (X_1, X_2) as the distinguished chief-factor pair. A direct product with an extra odd elementary abelian factor breaks this: take $PSL_2(32)$ with its order-five field automorphism and adjoin a trivial actor on C_7^2 . The simple chief factor has bottom group one, while its designated centralizer contains C_7^2 . Both printed alternatives can fail. Lifting the quotient argument controls a centralizer, not equality with the chosen bottom group.

Resolution. The corrected alternative replaces (X_1, X_2) by (X_1Y, X_2Y) . This retains the actual centralizer contribution instead of claiming that it was already contained in X_2 . The replacement proves the resulting disjunction and rebuilds the subsequent chief-factor comparison theorem from it. The signalizer application needs only the enlarged pair with the prescribed centralizer and therefore remains valid. The corrected theorem is used directly; the old alternative is not used. This is a statement-level correction accompanied by the necessary downstream proof changes.

Case 42. The smaller fixed subgroup has more than one maximal overgroup

Literature. Theorem 2.8(d), the $U_3(2^p)$ case, and its use in 6.2(d) [30].

Point of departure. For $U_3(2^p)$, the subgroup $C_1 \cong 3^2:2$ lies both in the small normalizer of 3^2 and in the much larger monomial subgroup. Hence the claimed unique maximal overgroup of C_1 is false. The full fixed subgroup $C \cong 3^2:Q_8$ does have the required stronger rigidity: its order-four action cannot preserve the same monomial configuration. The printed proof does not distinguish these two subgroups. Most uses involve C , but the later signalizer argument also needs information about C_1 , so simply replacing the symbol is not enough.

Resolution. The corrected statement gives uniqueness for C and replaces the later C_1 use by a prime-overgroup theorem. There is a prime r dividing the simple group order but not the order of the small normalizer such that every overgroup of C_1 containing an element of order r is the whole simple group. In the unitary case a prime divisor of $2^p - 1$ supplies this condition. The model-specific proof is assembled into the near-solvable signalizer theorem, providing exactly the generation consequence needed without asserting false maximal-overgroup uniqueness for C_1 .

Case 43. Several special 2-groups satisfy the printed recognition hypotheses

Literature. Lemma 2.6(a), p. 378, and its application in 3.19 [22].

Point of departure. The fixed-subgroup conditions on two commuting central automorphisms do not determine the special group of order 256. They allow several nonisomorphic groups, distinguishable by their involution counts; only one is the claimed central product of quaternion groups. The proof's assertion that each automorphism inverts the other's fixed subgroup is not a consequence of the stated conditions. Adding that inversion property isolates the intended group, but its use in Lemma 3.19 then requires a proof of the added property. The issue is a false recognition statement, not simply a missing choice of generators.

Resolution. The recognition shortcut is replaced by a finite classification of the actual semidirect products $P \rtimes V$, with $P \cong UT_3(\mathbb{F}_4)$. The configurations other than the intended quaternion shape make a specified subgroup characteristic and force the wrong Sylow containment, contradicting

the surrounding case assumptions. A finite computation supplies this exclusion to the proof of 3.19. The downstream $U_3(3)$ chain uses that corrected shape theorem and only those quotient facts that remain valid. This proves why the extra inversion behaviour holds in the application.

Case 44. The $SU_3(8)$ torus and the exceptional balance case

Literature. Part I, (20-7)(1) and the first claim (ii) of its proof [24].

Point of departure. For $G_2(8)$ at $p = 3$, a component $SU_3(8)$ contains a torus of order 57 and an element of order three normalizing its subgroup of order 19. Together with the central scalar, this actor is elementary abelian of rank two. It normalizes a nontrivial $3'$ -subgroup, contradicting the printed balance conclusion. The underlying claim that the relevant unitary group has only the primes two and three is true at field size two but false at field size eight. The exceptional component cannot simply be discarded from the list.

Resolution. The corrected theorem imposes an additional rank-three condition on the induced actor in the $G_2(8)$ case. That condition supplies the extra field action needed for the hyperplane argument to establish centralization. Other candidate ambient groups are treated separately by order: where 19 does not divide the group order, an $SU_3(8)$ component cannot occur. The proof of the later balance theorem uses this corrected statement, using either the verified rank hypothesis or the separate index argument in each application. It does not reuse the false unitary calculation.

Case 45. A quotient of order q^2 in a critical pair at distance two

Literature. (3.3)(i), the $b=2$ critical-pair index [31].

Point of departure. The point and hyperplane parabolics in $L_4(q)$ give the paper's own type of critical pair at distance two. Their unipotent radicals meet in a subgroup of order q , and the displayed quotient $Z_\alpha Q_{\alpha'}/Q_{\alpha'}$ has order q^2 , not q as asserted in (3.3)(i). Already $L_4(2)$ gives index four rather than two while the other local hypotheses hold. A citation to the preceding module calculation does not establish the index clause. Later arguments that only need the module or commutator structure must be distinguished from those that need the numerical index.

Resolution. The corrected version deletes the false index equality and separates the distance-two exceptional configurations. It proves corrected auxiliary alternatives for those configurations and uses them to obtain (3.6), while the later critical-distance arguments establish their needed bounds independently. In particular the proof of (4.3) has no remaining dependency on the rejected clause. The repair therefore changes both the statement and the arguments that use it. Merely weakening (3.3)(i) would not suffice wherever the original index had been used. The concrete $L_4(2)$ model remains as a check on the corrected statement.

Case 46. Quadratic fourgroups do not exclude the hexacode and tensor-module cases

Literature. Aschbacher–Smith, Vol. II, Lemma 13.8.21, hat subcases, pp. 956–957 [16, 17].

Point of departure. The proof eliminates the $3 \cdot A_6$ hexacode, $\widehat{M}_{22}$ and $L_3(2)$ hat configurations by placing a quadratic subgroup in a fourgroup and declaring this impossible. In the hexacode realization inside $SL_3(4)$, the relevant fourgroup consists of transvections with a common centre and is itself quadratic. The point-stabilizer fourgroup of $L_3(2)$ is also quadratic on its natural module and therefore on its tensor product with the natural S_3 -module. Thus neither the asserted contradiction nor the deduction that the quadratic actor has order two follows from the module structure. This is a failure of the exclusion argument, not merely a missing explanation of quadraticity.

Resolution. The replacement keeps the actual fourgroup $R = O_2(L_1T)$ and uses three finer facts: its nonidentity elements are not transvections on the relevant full section, its displacement has dimension at least four, and $|R : Q_H| \leq 4$. A rank-one image of U_γ would force displacement at most three. In the rank-two case the equality analysis of F.9.16 gives a two-dimensional quotient, while a commutator calculation forces a proper containment between two subspaces already of that dimension. The hexacode case is proved directly, the $\widehat{M}_{22}$ case reduces to it, and the tensor case receives its own argument. The auxiliary hexacode module assertion used in this reduction is supplied explicitly.

Case 47. The maximal-component comparison needs balancedness and an injective component count

Literature. Solomon, Maximal 2-components in finite groups, Lemma 3.2, pp. 574–576 [37].

Point of departure. The comparison in Lemma 3.2 assumes that a new centralizer satisfies the paper’s balancedness condition and that its lifted components can be counted as distinct. Neither follows from the displayed argument. A rank-two symplectic central-product configuration is overlooked in the balancedness step. In addition, distinct components before pumping up need not be counted as distinct afterwards without an injectivity argument. The final comparison of the third lexicographic quantity therefore cannot be justified merely by repeating the asserted component count.

Resolution. The replacement separates these obligations. It proves disjointness of the relevant sets of pumped components, supplies the missing rank-two symplectic kernel analysis needed for balancedness, and performs the last comparison using an actual Sylow-normalized product with the selected subgroup. These facts are then composed into the maximality argument. The proved scope is the simple, $\mathcal{K}$ -proper ambient setting required by the classification; this restricted version does not establish every unrestricted version of Solomon’s printed theorem.

Case 48. A commutator containment for HSS (3.6)

Literature. Huang–Stellmacher–Stroth, Proposition (3.6), claim (7) and the closing critical-distance argument, pp. 101–103 [28].

Point of departure. Claim (7) and the ending of (3.6) do not follow from the preceding block analysis. At critical distance five the required vertices may belong to the same local block; at larger distance a distant vertex can lie in a different block. The printed argument does not establish the commutator containment needed to pass between these possibilities. Reusing a pushing-up assertion from an earlier step does not fill the gap, because that assertion is itself too weak in the surviving configuration.

Resolution. The replacement proves directly that the relevant commutator $[V_\beta, V_{\alpha'}]$ lies in the specified earlier vertex group. The proof combines the natural-module chief factors, the nontrivial action bounds, and the rigid intersections of the actual local groups. This new containment yields a reduction to critical distance five. Both possible block positions are then analyzed, rather than assuming that one of them is impossible. Their contradictions establish the full natural-module case of (3.6). The required commutator containment is proved separately before it is used in the closing argument.

Case 49. An order estimate and module identification in HSS (4.3)

Literature. Huang–Stellmacher–Stroth, Proposition (4.3), odd critical distance, claim (7) and ending, pp. 106, 108–109 [28].

Point of departure. The claim that a trivial centralizer of the indicated perfect residual forces $|H| \leq 8$ is not justified; the available calculation allows order sixteen. The final step then identifies a quotient as an orthogonal S_5 -module although the local 2-core acts nontrivially in the remaining case. This module identification cannot be recovered from the smaller, unproved order bound. Both defects occur in the same odd-critical-distance branch and leave a genuine case after the printed argument ends.

Resolution. The replacement first proves that the relevant module has at most one noncentral chief factor. A separate module lemma then forces the local 2-core to act trivially on the quotient. This transfers the alleged remaining case to the other structural alternative and contradicts its defining assumptions. The small case with $|V_\beta| = 64$ and critical distance three is treated by an explicit finite-group calculation; larger distances use the revised critical-pair argument. Consequently the proof does not require the false order-eight bound or the unsupported orthogonal identification.

Case 50. The irreducible $E_9 : C_4$ alternative is not a product of dihedral groups

Literature. Aschbacher–Smith, Vol. II, Lemma 14.6.12(2), pp. 1030–1031, and Lemmas 14.6.13–14.6.17, through p. 1035 [16, 17].

Point of departure. The decomposition invoked in 14.6.12(2) misses the alternative $E_9 : C_4$ allowed by D.2.17. Irreducibility of the relevant 2-group action does not exclude this case. The following lemmas are then written for a product of two dihedral groups, so their arguments do not cover the missing configuration. In particular, one cannot apply those lemmas to $E_9 : C_4$ merely by reinterpreting its elementary abelian subgroup of order nine as the product in the printed hypothesis.

Resolution. The replacement separates the $E_9 : C_4$ configuration before entering the product-of-dihedral-groups argument. A global minimal-configuration analysis, using the bounds and structural information already obtained in 14.6.9 and 14.6.10, excludes this row. The large-2-subgroup case is treated with the corrected Meierfrankenfeld–Stellmacher pair theorem, applied to the actual pair in the global setting. Once this additional exclusion has been proved, the remaining alternatives do have the product structure required by 14.6.13–14.6.17, and those arguments can be used with their literal hypotheses.

Case 51. The Held case in the elementary-nine catalogue

Literature. Aschbacher–Smith, Vol. II, Proposition 1.3.4, pp. 509, 511–512; Lemmas 10.2.4, p. 747, and 15.2.12(2), p. 1111 [17].

Point of departure. The catalogue for an elementary abelian subgroup of order nine omits the Held group. Its automorphism group has the required local configuration inside a subgroup of shape $2^{4+4}.(S_3 \times S_3).2$; adjoining an external central involution also supplies the relevant ambient variant. Hence the omission is not resolved by checking the existing rows more carefully. Lemma 15.2.12 inherits the incomplete catalogue, while the local exclusion attempted in Chapter 10 fails in semidirect products with $\text{Aut}(\text{He})$. The two downstream uses require genuine additional work after the Held row is restored.

Resolution. The corrected catalogue includes the Held alternative. Chapter 15 propagates that row through its intermediate reductions and excludes it at the appropriate global stage using a small-commutator module bound: the available involution-conjugate generation, together with the order-seventeen obstruction, rules out the module demanded there. Chapter 10 instead derives the exclusion from the ambient hypotheses and the already established preceding reductions, with the later implication arranged so that it is not used circularly. The local Held examples are retained as valid examples; only their occurrence in the stronger global configurations is excluded.

Case 52. The S_8 and $U_3(3)$ cases in the small-module catalogue

Literature. Aschbacher–Smith, Vol. I, Lemmas G.10.15(3), pp. 361–363, G.10.16(4), pp. 363–364, and Theorem G.11.2, pp. 364–367 [16, 17].

Point of departure. The argument for G.11.2 uses two auxiliary assertions that fail in the S_8 configuration. Here $A_8 \cong \mathrm{GL}_4(2)$ can induce the full linear group on the relevant section, so the asserted properness of the induced group is false. Separately, $U_3(3) = G_2(2)'$ on its six-dimensional binary module has admissible subspaces in two orbits of length four; their existence does not give the invariant subspace used to exclude that row. Consequently both the S_8 and the $U_3(3)$ configurations are missing from the theorem’s list.

Resolution. The corrected auxiliary statement permits the exceptional rank-four linear image, and the classification explicitly adds the S_8 row and the six-dimensional $U_3(3)$ row. Their generation and subspace properties are verified in their actual modules rather than deduced from the false invariant-subspace argument. The revised proof then repeats the catalogue inspection with these alternatives retained, using the relevant small-group generation and module results for each row. Subsequent applications analyze the expanded list. The S_8 failures in the two auxiliary lemmas are treated as causes of the same missing-row error, not counted as independent classifications.

Case 53. Determinant twists of the $L_3(4)$ adjoint module

Literature. Aschbacher–Smith, Vol. I, Lemma B.4.14, p. 98, and Vol. II, Theorem 4.3.2, pp. 618–619 [16, 17].

Point of departure. The module list omits determinant twists of the adjoint module. Concretely, the 16-dimensional binary module obtained from $\mathfrak{sl}_3(\mathbb{F}_4)$ extends to the projective general linear group with an additional determinant character. The resulting modules have the same restriction to the derived group but need not have the asserted action of the diagonal extension. Therefore identifying the restricted $L_3(4)$ -module does not determine the full module required in B.4.14 or its use in 4.3.2.

Resolution. The corrected classification records the determinant parameter $k \in \mathbb{Z}/3\mathbb{Z}$ and proves the resulting eigenvalue and module alternatives. Each application must then determine whether the nontrivial twists are allowed. In the Chapter 11 use, a counting condition forces $k = 0$. The Chapter 4 argument cannot make that simplification, so it supplies a separate operator and composition-factor calculation for the twisted alternatives. This separates the classification of modules for the derived subgroup from their extensions to the larger group and closes the omitted case without identifying inequivalent representations.

Case 54. The $L_2(p)$ reduction lacks the required 2-core noncontainment hypothesis

Literature. Aschbacher–Smith, Vol. II, Lemma 10.2.5, the $L_2(p)$ step, p. 748 [16, 17].

Point of departure. The reduction in the $L_2(p)$ branch uses a conclusion that requires $O_2(X) \not\leq O_2(K)$, but that condition has not been established. The configuration with $p = 47$ shows that the local step cannot be applied as written. The remaining cases with $p \equiv \pm 3 \pmod{8}$ also retain an elementary abelian subgroup of order nine whose action must be controlled. Thus the subsequent exclusion is not justified by the local $L_2(p)$ identification alone.

Resolution. The replacement proves the precise elementary-nine conclusion needed in the application. It derives the required point-fixing information from the global hypotheses, the module theorem B.5.6, and the previously proved structural assertion 3.2.2(8), with the relevant A_4 configuration treated explicitly. This supplies the missing action information before the $L_2(p)$ branch is closed. The revised reduction does not assume the unavailable 2-core noncontainment or claim that the original local implication holds for every group with the displayed $L_2(p)$ section.

Case 55. Linear and field-extension cases in the rank-two amalgam list

Literature. Aschbacher–Smith, Vol. I, Theorem F.6.18, cases (6), (8) and (9), pp. 309–310 [16, 17].

Point of departure. The list in F.6.18 excludes actual amalgams supplied by $L_2(23)$ and by field extensions such as $\text{P}\Sigma\text{L}_2(25)$. These are not obtained by merely changing the notation for an existing case: the field automorphism changes the vertex normalizers, and the permitted local pair can have shapes involving $C_2 \times S_4$ and $S_3 \times D_8$. The printed proof therefore does not establish the asserted exhaustive list. The same omissions must be addressed wherever the theorem is used to reduce a global configuration to its listed amalgams.

Resolution. The replacement derives the cases from the applicable Goldschmidt amalgam classification while keeping the actual almost simple group and its outer field action. It adds the missing linear case and separates the field-extension alternatives instead of folding them into the original rows. The vertex groups and their common Sylow subgroup are checked with the extra automorphism retained. The subsequent Chapter 15 inspection is then rebuilt over this enlarged list, so the final exclusion does not rely on the false premise that all possible amalgams occurred in the original theorem.

Case 56. A proper arrow between local subgroups in pushing up

Literature. Aschbacher, Some results on pushing up in finite groups, Theorem 2, step (5.3), p. 68 [7].

Point of departure. Step (5.3) does not treat an arrow $Y \rightarrow Y^*$ with $Y \neq Y^*$. In the surviving case the upper quotient can be A_5 , while the lower quotient is either C_3 or A_5 . The printed argument proceeds as though the relevant natural-module configuration were already stationary. Neither the existence of an arrow nor the quotient label forces equality of the two subgroups, so the omitted transition is a substantive case in the pushing-up argument.

Resolution. The repair refines the arrow relation to include the natural-module data and chooses a maximal object above the original configuration with respect to that refined relation. A proper transition then forces the alternating natural-module case. The applicable sectional 2-rank theorem bounds the resulting section by four, contradicting the standing rank assumption

in the pushing-up setup. This excludes the proper arrow and justifies the stationary case used afterwards. The missing link is therefore derived from explicit module and rank information instead of being retained as a separate assumed recognition input.

Case 57. The triality argument proves a statement for a smaller subgroup, not for the original group

Literature. Timmesfeld, Finite simple groups in which the generalized Fitting group of the centralizer of some involution is extraspecial, (12.3), triality case, pp. 354–355 [43].

Point of departure. In the triality branch of (12.3), the proof passes from H to a subgroup built from $O^3(H')$ and the selected element, and establishes the desired structure only there. This does not establish the asserted conclusion for H . Groups of shape $\Omega_8^+(2) : S_3$ and $P\Omega_8^+(3) : S_3$ demonstrate the issue: the additional triality action changes the derived structure relevant to the statement. One cannot recover the original claim by silently identifying the smaller subgroup with H or by assuming that the original derived group is simple.

Resolution. The replacement keeps the full acting group and formulates the triality input in terms of the actual 3-local action. When the elementary abelian actor meets more than one coset of the relevant normal subgroup, the required fixed-point centralizer is proved nonsolvable with trivial 2-core. The classification is then applied to these actual alternatives, including the outer triality action, rather than to a subgroup for which the conclusion is easier. This supplies the structural information used later while avoiding the unsupported assertion about H' in the original reduction. The proof is not a verification of the stronger printed intermediate assertion.

Case 58. The $\mathrm{Sp}_6(2)$ spin module in the width-four catalogue

Literature. Timmesfeld, *Finite simple groups in which the generalized Fitting group of the centralizer of some involution is extraspecial*, (12.3), (12.12), (12.14) and the width-four analysis, pp. 354–355, 359–360 [43].

Point of departure. The alternatives in the width-four analysis omit $\mathrm{Sp}_6(2)$ acting on its binary spin module. This configuration is not one of the printed rows under a different label, and the earlier local arguments do not exclude it. As a result, the conclusions of (12.3) and their subsequent use in (12.12) and (12.14) are incomplete. A valid repair must distinguish adding the missing local alternative from proving that it cannot occur in a particular later global application.

Resolution. The corrected Section 12 catalogue carries an explicit spin-module alternative through the relevant inspections and into the later structural reduction. Where Section 14 needs to rule it out, a separate comparison of the actual 2-parts of group orders excludes that configuration under the stronger hypotheses available there. The proof therefore does not force a local contradiction from data that allow the spin module. It first classifies the complete set of possibilities, then performs the additional global exclusion at the point where its hypotheses have been established.

Case 59. The order of the distinguished lift is not established by the printed commutator claim

Literature. Aschbacher, The Tits group as a standard subgroup, (5.16)(2) and the proof of (5.18), p. 243 [10].

Point of departure. The proof requires z_0 to have order two, but the preceding data still allow $z_0^2 = z$. The later calculation uses a commutator containment for all of P_{10} that has not been proved and would impose an action inconsistent with the subsequent configuration. Therefore neither treating the lift as an involution nor appealing to the printed containment supplies the needed missing step. The distinction is significant: the order of a lift is not determined by its involution image in a central quotient.

Resolution. The repair isolates the smaller commutator statement actually needed, involving the appropriate intersection $V \cap N$ rather than all of P_{10} . It reduces that statement to explicit local calculations in the two possible centralizer images and provides those calculations separately. The resulting containment feeds a square calculation that proves $z_0^2 = 1$. Thus the corrected route establishes the involution lift before using it and does not retain the overly strong action claim. A later written argument also explains the obstruction through the equivariant commutator pairing; the repair used here is the two-case containment and square analysis. The broader printed commutator assertion is not proved.

Case 60. The rigid-edge case of the classical involution theorem

Literature. (28.2), Case B2b [4].

Point of departure. The printed treatment of (28.2) does not dispose of Case B2b. This is the rigid-edge alternative in the local link analysis, so completing the other shapes does not remove it: the relevant fourgroup and its Sylow position still have to be controlled. There is no counterexample to the full theorem in the evidence; the defect is the absence of an argument for a genuine branch of its case division.

Resolution. The replacement refines the Sylow configuration. In the residual B2b form, the designated fourgroup $E = \langle t, u \rangle$ is abelian and strongly closed in the relevant Sylow 2-subgroup. Goldschmidt's theorem, followed by the previously established (17.8) exclusion, eliminates that possibility. The complementary B2a form is handled by the frame argument and the theorem at smaller order. Together these arguments settle the rigid-edge case with the original induction assumptions explicit. The repair supplies the omitted local alternative, not a new blanket assertion that every fourgroup in such a link is strongly closed.

Case 61. The twin-thin cross-pair argument

Literature. (12.11)(2), the twin-thin case with $z \notin O_2(G)$ [3].

Point of departure. The proof of (12.11)(2) does not establish the twin-thin conclusion when $z \notin O_2(G)$. It is insufficient to know the abstract type of each local factor: one must show that the prescribed pair in the original frame is a thin cross-pair. The missing placement argument also affects whether the smaller group used for induction has the required odd-core and transitivity properties.

Resolution. The replacement chooses an involution u in the appropriate normal 2-subgroup of a conjugate chart and forms the layer generated by the family inside $C_G(u)$. Odd-core balance, using the vanishing of the relevant involution-centralizer odd cores and coprime generation, proves that this layer has trivial odd core. Its transitivity and the hypotheses of the induction theorem are then checked explicitly. The resulting alternatives are treated separately in the small degrees five and six and in larger degree. Comparing the actual four-set labels and twin positions supplies the cross-pair conclusion in the original frame, rather than merely recognizing an isomorphic local model.

Case 62. Strongly closed subgroup analysis replaces the normalizer inspection

Literature. Chapter 9, Lemma 4.15, assertion (4G), p. 408; its use in Proposition 4.12(a) and Lemma 8.9 [25].

Point of departure. Assertion (4G) puts the relevant subgroup Y in $N_G(E)$ by inspecting $C_G(x)$. The difficulty lies in a different involution centralizer: the requisite balance modulo its odd core is not a consequence of the displayed structure of $C_G(x)$. Thus the statement is not supplied by the inspection as cited, and later use of Proposition 4.12(a) cannot erase that missing normality step.

Resolution. The replacement applies Goldschmidt's strongly-closed abelian subgroup theorem directly in the actual local setting. With the established bound on the Sylow 2-subgroup and the presence of an involution outside E , it proves that $E O_{2'}(C_G(v))$ is normal in $C_G(v)$. The relevant alternatives then make this centralizer soluble in the designated small row, or place it in the intended subgroup M in the alternating case. These are exactly the local conclusions required downstream. The route does not assert a general inspection principle for Y , and it retains the separate known-group inputs to Goldschmidt's theorem.

Case 63. The alternating signalizer argument omits diagonal fourgroups

Literature. Lemma 3.6, the two-point centralizer case for $n = 9$, pp. 544–547 [39].

Point of departure. To prove $C_G(f) \leq H$ for a two-point actor, the argument uses the action of a rank-three elementary abelian group $B = E \times \langle t \rangle$ on an odd subgroup. There are seven fourgroups in B , but the printed construction controls only four of them. The other three are diagonal fourgroups, not members of the original signalizer family and not containing t . Coprime generation requires their centralizers too, so the desired odd-core containment does not follow from the four controlled terms.

Resolution. The replacement states the missing diagonal balance condition explicitly and proves the local normalizer containment when it holds, always using the original signalizer subgroup. If the condition fails, L -balance places the obstruction in an actual component of the complementary local layer; this is analyzed as a separate residual configuration. In particular, the six-point

dihedral case is treated independently instead of being hidden in the generation formula. The two branches are then supplied to the odd-degree alternating theorem. No universal assertion that every diagonal fourgroup belongs to the original family is made.

Case 64. Strong embedding does not imply equality with the ambient group

Literature. Theorem 2.6, p. 534, and the $m = n + 4$ conclusion of Lemma 3.6, pp. 545–546 [39].

Point of departure. Theorem 2.6 is used to conclude $H = G$ from centralizer and fusion control. Its asserted form is too strong: the familiar pair $A_4 < A_5$ already shows why such control can lead to a proper strongly embedded subgroup. The cited theorem establishes strong embedding under its additional hypotheses, not equality with the ambient group. Consequently the final sentence of the $m = n + 4$ case cannot simply invoke the printed Theorem 2.6.

Resolution. The replacement first proves the required centralizer containment for the actual involution w , using the relevant recognition theorem and explicitly excluding its small $L_2(4)$ and $U_3(4)$ alternatives. It then applies the valid strongly-embedded-subgroup result to H . Strong embedding gives a single ambient involution class in the relevant situation; the already constructed eight-point involutions yield an incompatible class. This provides the contradiction that the printed equality shortcut omitted. The proof does not assume that w is Sylow-central for every degree, and it does not replace strong embedding by the false universal conclusion $H = G$.

Case 65. Constructing the required lift in degree nine

Literature. Lemma 3.6, $m = n = 9$ and the eight-point case, p. 547 [39].

Point of departure. The degree-nine proof chooses a lift f with a specified permutation action and square, and uses it to obtain $\rho(H) \leq A_9$. The required lift does not follow just from its image in a symmetric group: its square can be the central involution, and several possible centralizer configurations remain. If the image were only contained in S_9 , the relevant normalizer quotient would have order 72 rather than 36, so the final counting contradiction depends materially on the unproved choice.

Resolution. The replacement separates lift existence from the final count. It classifies the relevant permutation-centralizer configurations and combines that finite analysis with the independently proved uniqueness of the two-point centralizer. This produces a lift with the actual required square and action, thereby establishing the alternating-image conclusion. Only after that conclusion is available does the proof use the elementary abelian subgroup of order 32 and its normalizer to finish the fusion count. The finite check is carried out directly; a count performed merely in an abstract A_9 model is not substituted for the lift.

Case 66. Being outside the 2-residual does not determine the alternating support

Literature. Lemma 3.6, Case 1, the four-point inference, p. 545 [39].

Point of departure. The proof infers four-point support in the alternating image from an involution lying outside the relevant O^2 subgroup. This need not hold if the involution has a nontrivial component in another factor. For example, an eight-point involution in the alternating factor can be multiplied by an involution in an independent A_5 factor; the latter centralizer changes the 2-residual test without changing the eight-point alternating image. Thus the quotient support cannot be read off from that membership test alone.

Resolution. The replacement does not use the unrestricted implication. It formulates a narrower assertion for the precise involutions constructed in the two applications in Case 1 of Lemma 3.6, retaining their origin and their actual permutation action. The remaining configurations are excluded by a case analysis on the shape of $\rho(s)$, with the eight-point possibility handled separately. This is sufficient for both applications and avoids assuming an unproved global restriction on the complementary layer. The repair therefore proves that the bad configuration cannot arise in the construction; it does not establish the unrestricted implication.

Case 67. Component analysis for induction on an involution-centralizer quotient

Literature. Lemma 5.2, outer-automorphism case [38].

Point of departure. The argument applies induction to $X = C_G(v)/\langle v \rangle$. It does not establish that the required component is standard in X , that its centralizer has the required rank, or that the relevant involution centralizers in X have trivial odd core. A later application involving the Higman–Sims case has the same problem. These are structural hypotheses of the recognition theorem, not immediate consequences of taking a central quotient of an involution centralizer.

Resolution. The replacement performs a proper-section pump-up analysis before invoking recognition. It tracks the image of the actual component through the quotient, proves the needed fixed-layer and centralizer properties, and checks the odd-core conditions in the available known-group alternatives. The Higman–Sims branch is reached only after those checks establish its own hypotheses. The Section 5 argument uses the resulting table of component configurations and the established induction hypotheses. It does not assume that standardness is inherited by arbitrary centralizer quotients. This retains the original ambient simple-group restrictions and supplies the particular quotient configuration actually used.

Case 68. An $L_3(2)$ -module lifting argument in place of transfer

Literature. Lemma 4.6, p. 171 [38].

Point of departure. The lemma must distinguish $T \cong C_2^6$ from $T \cong C_4^3$. Its proof uses a transfer hypothesis excluding certain conjugates of t from a derived normalizer, and also claims that a subgroup denoted SQ^n is Sylow. Neither statement follows from the stated fact that t is not conjugate into the original component. Without them the transfer conclusion eliminating the exponent-four alternative is unsupported.

Resolution. The replacement proves the elementary-abelian conclusion by following the $L_3(2)$ module structure through successive normalizer levels. The starting configuration, the step to the next level, and each terminal case are stated separately, so the Sylow positions and the action on the relevant elementary sections are checked at the point of use. The exponent-four alternative cannot persist through this iteration. Together, these lemmas for successive normalizer levels and their finite checks give the original conclusion $T \cong C_2^6$, without introducing the printed transfer or Sylow assertions as unexplained hypotheses.

Case 69. A product case in the $GL_6(2)$ overgroup list

Literature. Lemma 4.7, overgroups in $GL_6(2)$ and the second proof paragraph [38].

Point of departure. The asserted overgroup list omits $L_3(2) \times S_3$. Such a group occurs in the relevant six-dimensional representation and can be seen in a suitable parabolic configuration in $L_5(2)$. The next proof paragraph also excludes small alternating alternatives without supplying the claimed argument. An abstract list of familiar simple overgroups is therefore insufficient: the actual group need not be almost simple, and its Sylow position is part of the problem.

Resolution. The replacement uses a complete overgroup census for the actual involution-centralizer configuration inside $GL_6(2)$, retaining the product possibility. It then proves a separate local order statement: the previously constructed TS_0 is not Sylow in $N_G(T)$, which forces a 2-part of at least 2^{11} in that normalizer. Comparing this with the corrected finite alternatives removes the small candidates that the printed paragraph had discarded prematurely. The census, the Sylow-order bound, and the final exclusions are separate parts of the proof. Correcting the list alone does not complete the argument.

Case 70. A normalizer-tower contradiction replaces the unproved larger standard component

Literature. Lemma 4.10, pp. 172–173 [38].

Point of departure. The final Section 4 argument constructs two groups M_1, M_2 , asserts that the relevant generated subgroup is their direct product, and proceeds as though M_2 were standard. The necessary generation and standardness statements are not proved. Neither follows from the abstract types of M_1 and M_2 , because the ambient conjugacy and centralizer conditions are essential. Thus the final recognition step is not justified merely by the preceding local structural description.

Resolution. The replacement starts directly from the local structure already established in Lemma 4.9. It analyzes the normalizer tower of the relevant Sylow 2-subgroup and applies an extremal Thompson-transfer argument to its distinguished subgroup. The wreath and linear configurations are handled separately, and each yields a contradiction to simplicity. This produces the required exclusion without constructing $M_1 \times M_2$ or asserting that M_2 is standard. The

lemmas for the normalizer tower, together with the required results for the finite models, replace the original generation assertion in the Section 4 proof.

Case 71. Small-rank-one families in the class-two Sylow classification

Literature. Theorem 2.4, pp. 502–503 [40].

Point of departure. The quoted classification of characteristic-2-type groups with class-two Sylow 2-subgroups and the stated rank condition leaves only the indicated linear and symplectic families. As printed, it also admits groups such as $Sz(32)$ and $U_3(16)$, which are not on that list. The exclusion of rank-one or strongly-embedded configurations needs an additional local hypothesis. The mere class-two and rank assumptions do not provide it.

Resolution. The replacement pushing-up argument avoids the quoted classification. It retains the actual nonsoluble maximal 2-local subgroup supplied by the application and uses the detailed local analysis from Gomi’s Sections 3–4 together with the relevant Sylow-structure theorem. From that stronger setting it proves the required symplectic Sylow conclusion and the subsequent conclusion of printed (5.5). This is an application-specific bypass, not a proof of the original incomplete list. In particular, no claim is made that $Sz(32)$ or $U_3(16)$ fail the printed class-two hypotheses.

Case 72. The M_{12} branch needs real-element fusion instead of a signalizer

Literature. (15.13), the M_{12} case, pp. 119–120 [11].

Point of departure. In the M_{12} case, the relevant elementary abelian 3-group together with the designated involution does not normalize a nontrivial 2-subgroup of the required kind. The printed signalizer argument therefore cannot be applied. This is a specific failure of its local mechanism, rather than an omitted numerical verification in a general signalizer theorem.

Resolution. The replacement uses the fact that all elements of order three in M_{12} are real, with the inversion lifted through the actual $3'$ -core. Writing $A = B\langle c \rangle$, the construction chooses an element $a = yd$ with $y \in B \cap K$ and $d \in \langle c \rangle^\#$. An inverter in K , combined with an element in $L \cap N_G(A)$ that inverts B , conjugates a to d . Uniqueness of the appropriate actual component then forces $K^g = L$. Hence the quotient of L would have type M_{12} , contrary to the previously established list. No nonexistent normalized 2-subgroup is required.

Case 73. The asserted intermediate subgroup misses outer and covering cases

Literature. (3.15), proof, p. 57 [11].

Point of departure. The proof selects a subgroup H containing the specified Sylow 2-subgroup with quotient $L_2(2^n)$ or S_3 , and then uses $G = \langle H, N_G(S) \rangle$. The proposed selection is not available uniformly: $L_2(16).2$ and $3.M_{10}$ expose the missing outer-automorphism and covering possibilities. These groups invalidate the intermediary construction, even though the desired characteristic-subgroup conclusion can still hold.

Resolution. The replacement treats the possible quotients separately. The alternating, small linear and $3.A_6$ cases use the appropriate small-group local theorem; a split central factor is handled directly when the Sylow subgroup is already normal. For the larger even-field linear cases, the proof distinguishes whether the Sylow subgroup contains the necessary part of $O^2(X)O_2(X)$. In one branch a local component/block theorem supplies the desired characteristic subgroup; in

the other, Thompson factorization or centralization of $\Omega_1 Z(O_2(X))$ does so. This proves the conclusion of (3.15) needed in the application without assuming the false uniform subgroup H .

Case 74. The Section 11 uniqueness argument is bypassed to break a circular dependence

Literature. (11.12), first half, and the uniqueness claim in (11.13)(3) [11].

Point of departure. The first half of (11.12), needed for uniqueness in (11.13)(3), has no supplied proof. Attempting to recover that uniqueness from later Section 11 lemmas produces a circular dependence: those lemmas pass through a result which already uses (11.13). Thus neither a citation to the subsequent case analysis nor an assertion of the desired uniqueness supplies the missing argument.

Resolution. The replacement argument avoids this uniqueness statement. The applications in the later local case analysis are replaced individually. In the crucial (11.19) branch, the possible point types are derived directly from the standing configuration and the existing local component restrictions, rather than from a chosen line whose uniqueness depends on (11.12). The other applications use narrower statements established under their own hypotheses. This proves the required Section 11 conclusions without circular reasoning. It does not prove the unused first half of (11.12) in its printed generality.

Case 75. Different commutator subgroups are needed in the uniqueness argument

Literature. (22.6), (22.7), and the final proof of Theorem 22.2, pp. 535–537 [12].

Point of departure. The proof sets $E = \Omega_1 ZJ(T)$, $S = C_T(E)$ and forms $E_i = [F_i, S]$ from the indicated conjugate-generated groups. It treats the E_i as nontrivial 2-groups and invokes the preceding normalizer theorem at $N_i(S)$ without proving its hypotheses. Local models based on S_4 show that such commutators can have nontrivial odd part, so the missing verification cannot be supplied by an abstract commutator identity. The closing contradiction consequently lacks the required legitimate 2-local subgroups.

Resolution. The replacement uses the block commutators $\hat{E}_i = [I_i, J(T)]$. It proves that these are the required nontrivial 2-subgroups and establishes their normalizer containments with the actual hypotheses on the blocks checked. The noncyclic odd-prime configuration then gives $N_G(\hat{E}_i) \leq M$; the strong embedding information supplies the necessary containment for $N_G(S)$. A characteristic common subgroup completes the normalizer contradiction. The Section 22 proof uses these replacement commutators throughout. It does not claim that the original $[F_i, S]$ satisfy the false 2-group assertion.

Case 76. Odd-core balance in place of a two-generated-core equality

Literature. (23.8), the Z^* case and its signalizer step, p. 435 [4].

Point of departure. The signalizer step asserts that the involution centralizer equals the subgroup generated by the relevant rank-two normalizers. In $Sp_6(2)$ that subgroup can have index three, including the prescribed elementary abelian configurations. Thus the argument cannot invoke the earlier generation theorem to kill the signalizer in this manner. The counterexample concerns the intermediary generation claim, not the full conclusion of (23.8).

Resolution. The replacement proves that the preliminary signalizer subgroup α is normalized by both of the actual involution centralizers $C_G(u)$ and $C_G(t)$. Those centralizers generate the ambient group in this configuration, so the required proper odd subgroup must be trivial. With $\alpha = 1$, Part I (3.2.5) applies to the commuting involutions and their action on quotient components, giving equality of the two odd cores. That common odd core is normalized by both generating centralizers and is therefore trivial. The proof reaches the same local conclusion without the false two-generated-core equality.

Case 77. An intersection is treated as an automorphism centralizer

Literature. (10.9), the $L_2(8)$ paragraph, p=7 [6].

Point of departure. The argument treats an element b as normalizing the component of $C_G(x)$ under discussion, although b does not centralize x and need not normalize that component. Hence the intersection with $C_G(b)$ is not the centralizer of an automorphism of the component. In $SL_6(2^7)$ extended by its field automorphism, the asserted local data occur and the relevant $SL_2(2^7)$ sits diagonally across two components. The printed attempt to place it in one component fails in this model. The model is excluded by global hypotheses that this paragraph does not use.

Resolution. The replacement uses fusion and transfer at the prime seven. It establishes the requisite field-automorphism fusion data in the actual setting and applies the global transfer argument to obtain a normal subgroup of index seven. Simplicity rules this out. This also completes the remaining recognition argument, which therefore does not rely on the defective $L_2(8)$ local step. Unlike the printed calculation, the replacement uses the ambient condition that distinguishes the excluded configuration from a genuine Lie-type group extended by a field automorphism.

Case 78. A square acting as a field automorphism does not fix every torus line

Literature. (12.15), final symplectic graph-automorphism case [6].

Point of departure. The exceptional graph automorphism of $Sp_4(q)$ interchanges the two root lengths. Its action on torus coordinates can be represented by a matrix whose square is $2I$; its square acts as a power map, while the automorphism itself does not preserve every line. Thus g^2 being a field automorphism does not show that g normalizes the chosen root component or the torus used in the last paragraph. The asserted product decomposition of that torus, and the inversion contradiction based on it, are unsupported.

Resolution. The replacement works with a characteristic subgroup $Q = \langle x \rangle \times \langle y \rangle$ of an actual Sylow p -subgroup. A Frattini argument supplies two normalizer lifts: one moves a prescribed line and fixes the field actor modulo the torus, while the other would invert that actor. Both act on the same Q , so integer-matrix and pointwise commutator relations apply simultaneously. They

force $r \equiv r'$ and $r \equiv -r' \pmod{p}$ with $p \nmid r$, impossible for odd p . This excludes the inverting element without constructing the unjustified Cartan product.

Case 79. Quartic monomial reflection groups in the case division

Literature. (13.21)(2) and (13.22)(1), use of the monomial case of (3.12) [6].

Point of departure. For primes $p \equiv 1 \pmod{4}$, the groups $G(4, 4, 3)$ and $G(4, 2, 3)$ are irreducible monomial reflection groups over $\mathbb{F}_p$. Their orders are 96 and 192, and products of their reflections have only the orders allowed by the printed local restrictions. They are neither the displayed A_3 nor B_3 Weyl group. The printed use of Bloom's list omits these possibilities, so the inference to the claimed reflection action is incomplete. This does not exhibit an ambient counterexample to the rank-three theorem; it exhibits cases that the cited classification alone does not exclude.

Resolution. The replacement includes the full monomial alternatives. In the first frame, the actual types of the attached rank-one groups and the associated fusion restrictions exclude $G(m, m, 3)$. In the second frame, a separate condition on the distinguished reflections forces the diagonal entries to be signs, leaving only signed permutation matrices. The known rank-two subgroup and transitivity then give the whole $W(B_3)$ action, rather than merely an upper bound. These arguments form the monomial branch of the repaired (13.22), with the exceptional symplectic row handled by its separate root-group theorem.

Case 80. The central SU_5 case

Literature. (12.5), the SU_5 branch and the use of (10.5) [6].

Point of departure. The proof first obtains a component in a broad family, but applies later results that require its narrower distinguished subfamily. The missing membership is not supplied by the cited form of (10.5). In addition, when the component is $SU_5(q)$ and the relevant element of order five generates its centre, the claimed contradiction disappears: the produced group can lie entirely in its centralizer. This is a real unhandled branch of the proof, not a reason to discard central covers. The statement of (12.5) itself is not refuted.

Resolution. The replacement strengthens the component-selection step to the distinguished family, using the actual invariant 2-group construction. It then obtains the required rank-two membership, constructs a transvection in the relevant induced action, and contradicts the independently established no-transvection theorem. The SU_5 possibility is treated explicitly in this argument. The argument uses a separately proved form of the result producing a transvection, avoiding a circular appeal to the subsequent printed arguments. This replaces the appeal to (11.1) that lost the central case while retaining the original desired conclusion.

Case 81. A bounded comparison of component configurations

Literature. Lemma 3.4(3), commuting-component case, pp. 410–411 [9].

Point of departure. The proof replaces a tuple of involutions and components by a new tuple and then invokes the preceding configuration again. It neither verifies all of the new tuple’s hypotheses nor exhibits a decreasing parameter. Some of the new normalizer and component-containment conditions are precisely what must still be proved. The argument therefore cannot be read as a well-founded descent. The first branch also needs the corrected version of an earlier two-involution lemma. This gap propagates to the later standard-component comparisons; no counterexample to the statement is established by it.

Resolution. The replacement bounds the comparison to three explicit levels. At each level it specifies the components, the distinguished involutions, and the commutation conditions. Two additional lemmas provide the missing transitions, using the corrected earlier statement and the appropriate component theorem, rather than presuming that a replacement tuple has the same properties. The final level gives the contradiction directly, so no infinite repetition or informal minimality is required. This establishes the commuting-component comparison required in the recognition argument in characteristic three.

Case 82. The inner cyclic-four centralizer case

Literature. Lemma 6.3 and the preliminary bound in 6.4(i) [9].

Point of departure. In $\Omega_6^+(3)$ an inner involution can have an anisotropic two-dimensional minus-space. A rotation of order four on that plane, combined with the appropriate action on its orthogonal complement, lies in the group and centralizes the $L_2(9)$ component. The relevant centralizer quotient can therefore be cyclic of order four, not of order at most two as printed. The resulting $C_2 \times C_4$ centralizer does not force the ambient 2-rank to be two. The contradiction at the end of 6.3 consequently misses an actual local possibility.

Resolution. The replacement starts with the actual bad pair and its arrow into $\Omega_6^+(3)$. Applying component balance at two common centralizers produces another $L_2(9)$ component. The possible component arrangements force a subgroup of order at least four in its intersection with a centralizer that, by an explicit orthogonal calculation, has 2-part only two. This contradicts the configuration directly and treats both the small and cyclic-four branches. The subsequent alternatives retain the previously omitted case until this argument applies; the unsupported assertion about arbitrary 2-groups is never used.

Case 83. The involution type $i(6, -)$

Literature. Lemma 2.14(2), final clause, and its use in Lemmas 4.5–4.7 [9].

Point of departure. The joint centralizer condition in 2.14(2) does not force the involution type $i(2, -)$. Eigenspace decomposition in the seven-dimensional orthogonal model also permits $i(6, -)$: the five-dimensional common eigenspace still supplies the required $\Omega_5(3)$ component. The second paragraph of 4.5 invokes precisely the false exclusion. Further local models show that the desired fusion statement cannot be extracted solely from those centralizer calculations. The correction must distinguish a false local inference from the global simple-group statement, which has additional simplicity and fusion hypotheses available.

Resolution. The replacement proves 4.7 first and recovers the needed part of 4.5 afterwards. It analyzes the actual involution frame and separates all support types, retaining the outer cases that genuinely occur locally. Suitable sign characters provide an index-two subgroup avoiding the relevant fusion class. If the first normalizer does not contain a Sylow subgroup of the ambient group, the argument controls the one-step Sylow enlargement and its fusion before applying Thompson transfer. This uses simplicity at the point where local orthogonal geometry alone was insufficient and removes the false type exclusion from the recognition route.

Case 84. Component containment is used before its commutation hypothesis is known

Literature. Lemma 5.13, appeal to Theorem 2 of Aschbacher (1975) [9].

Point of departure. The cited component theorem can place $K(t)$ inside the component at a neighbouring involution only after the relevant commutation is known. That commutation is essentially the next lemma, which is itself derived from 5.13. The special neighbouring involution $s = tz$ makes the issue particularly visible: the centralizer layer to which the citation is applied can be trivial, so the theorem gives no containment. The actual model satisfies the intended conclusion, but the printed citation does not establish it. The circularity concerns the hypotheses of the application, not a counterexample to the cited theorem.

Resolution. The replacement first proves the required commutation with the 2-core directly in the elementary abelian model, using explicit automorphism information for $\Omega_6^-(3)$. A separate component-comparison result then gives the base case of 5.13 for the prescribed neighbours, and the argument extends that conclusion to the whole neighbour set. This supplies 5.13 before 5.14 is invoked. The required facts about the model and the balance properties are stated explicitly. In particular, the initial commutation statement is proved before the lemma that depends on it.

Case 85. Two rank-two 3-subgroups in $E_6(2)$ are not conjugate

Literature. Lemma 2.44, last assertion, and its use in Lemma 6.6(iii) [32].

Point of departure. In the A_1A_5 subsystem of $E_6(2)$, let z have order three in the A_1 factor and let y be the scalar of order three on the $\mathbb{F}_4^3$ realization of the six-dimensional module. The common component is $J = SL_3(4)$ and $D = \langle z, y \rangle$. The element z has algebraic centralizer dimension 36, whereas order-three elements of J have centralizer dimensions 24 or 30. Hence D cannot be conjugated into J . The later proof cannot transport its required normalizer conclusion along the asserted conjugacy.

Resolution. The replacement proves the required $C_G(J)$ containment from actual $E_6(2)$ model data. It treats the line generated by the central element of J separately from the other lines of D , and uses conjugation by elements normalizing the actual common component and the relevant root subgroup for those other lines. Root closure, p -core and invariant-subgroup calculations then supply the needed normalizer properties. This dedicated exceptional argument is used at the original point of application. It never identifies D with the torus intersection in J , so the false conjugacy plays no role.

Case 86. The amalgam does not supply the symmetry needed for an offender

Literature. (5.12), step 5 of the failure-of-factorization argument [31].

Point of departure. The two displayed commutator bounds produce an offender on whichever side has the favourable index inequality. The proof implicitly interchanges the two adjacent vertices to choose that side. In the amalgam with local group $T \text{Aut}(T)$, an element sending one neighbour to another need not swap them: its square need not lie in the edge subgroup. The transported bad-pair data can therefore belong to a third neighbour. The printed symmetry argument does not identify the claimed module at the original vertex, so its appeal to the failure-of-factorization theorem is unjustified.

Resolution. The replacement pairs the two inequalities without assuming a vertex swap and extracts the required offender on the correct common quotient from either orientation. For larger fields the corrected failure-of-factorization classification then excludes the configuration. At $q = 2$ the proof treats the $3 \cdot A_6$ possibility by an isotropic-plane argument and the A_6 possibility by explicit $Sp_4(2)$ coordinates. It proves the core conclusion of (5.12) without the intermediate natural-module assertion used in print. The proof of the final theorem uses the corrected core conclusion and does not require the stronger natural-module assertion.

Case 87. An additional term in the centralizer-index estimate

Literature. (4.5.3)(i), the D_0 argument at $q=2$ [31].

Point of departure. The proof bounds the action of $\widehat{V}_{\alpha-2}/H_{\alpha-2}$ by q using information that controls only a quotient of a subgroup. The group $Z_{\alpha-2}$ already contributes nontrivially to the action and is not killed by $H_{\alpha-2}$: its critical-pair commutator lies outside that subgroup. This contribution is omitted from the estimate. At $q = 2$ a larger-field torus argument cannot repair the omission, since the relevant torus is trivial. The defect is in the proof of the required inclusion, not an established counterexample to the final amalgam theorem.

Resolution. The field-two repair constructs a second good path branching at the original vertex and compares the two sets of orbit generators. This proves the missing inclusion $Z_{\alpha} \leq V_{\gamma}^{\circ} Z_{\gamma}$, from which the needed statement about $\widehat{V}$ follows with the already established invariance properties. The proof does not bound the entire action by ignoring the $Z_{\alpha-2}$ contribution, and it does not assume a splitting that fails for the small module. This repair establishes the field-two inclusion used at this point. It does not supply the other ingredients of the subsequent argument.

Case 88. One element need not realize two simultaneous swaps

Literature. (4III.7), the case $|T|=8$, $|L|=4$ [42].

Point of departure. The argument chooses an element of T that is supposed to imitate an automorphism on two pairs of conjugate subgroups simultaneously. Elements outside L swap both pairs, but the displayed hypotheses allow the automorphism to swap one pair and fix the other. On the associated two-dimensional space over $\mathbb{F}_3$, these are respectively scalar inversion and a reflection, so they cannot agree. An explicit semilinear unitary model realizes the mixed local action. This invalidates the printed local step; the global weak-closure hypothesis must be used to exclude the case.

Resolution. The replacement applies weak closure to the conjugate T_x containing the offending involution x . It studies the elementary abelian commutator group $[x, N]$, rather than searching for a single element that imitates both swaps. In the mixed case this group has order at least eight and would act faithfully on $T_x \cong C_2^3$. The action constraints force an impossibility in that small linear group. This separate local theorem supplies the order-eight case to the main proof. The correction retains the original statement but replaces the simultaneous-swap argument with a use of the hypothesis that the local model does not satisfy.

Case 89. A maximum over central involutions is used for arbitrary involutions

Literature. (8.1)–(8.2), their common final fusion step [42].

Point of departure. In (8.1) the maximization ranges over central involutions of a Sylow subgroup, but the final comparison uses an arbitrary involution of that subgroup. The inequality does not follow from the chosen maximum. In (8.2) the analogous step depends on the assertion that a subgroup is generated by conjugates of the original TI-subgroup, although only individual elements of those conjugates have been placed there. Weak closure gives normalization, not the missing containment. The two arguments therefore do not establish the fusion statement on which the final contradiction rests.

Resolution. The replacement uses the connected component of the original nonidentity TI-set in the commuting graph. Its generated subgroup has precisely the closure properties required by the fusion argument, and these are proved before the induction is applied. This supplies

the missing condition in the final step of (8.2). A separate argument gives the condition for (8.1). The rank-one endings use Thompson transfer and Glauberman's Z^* theorem with their actual hypotheses. These theorems complete the final argument without extending the maximum beyond its stated range or relying on the unproved generation assertion.

Case 90. The component used in the contradiction may act trivially

Literature. (4I.5), exclusion of a component in the local quotient [42].

Point of departure. The cited lemma excludes a component only when that component acts nontrivially on the relevant elementary abelian group T . The printed proof does not establish this. A normal 2-subgroup can act nontrivially inside the intersection of the conjugates of L while acting trivially on the quotient; transitivity on $T \setminus L$ does not eliminate that kernel. With two components, nothing identifies the particular factor whose action is nontrivial. Thus the existence of a component alone is insufficient for the proposed contradiction, and the overlooked trivial-action branch must be handled.

Resolution. The replacement sets $A = \bigcap_g L^g$ and works on T/A , where the normal 2-subgroups have the needed trivial action. If the product of the components acts trivially there, then $L = [T, L_1] \leq A$, making L normal in the local normalizer; the odd-transposition argument gives the contradiction. Otherwise a specific component acts nontrivially on the quotient, so the cited action lemma applies to that component and supplies the required rank bound. The no-layer theorem covers both branches and is used by the main TI-subgroup proof.

Case 91. A vertex-fixing product is not automatically central in an edge stabilizer

Literature. (5.14), the perfect-position and fixed-arc steps, pp. 396–397 [21].

Point of departure. The proof applies perfect-position lemmas to labels attached to neighbours off the distinguished line, although those lemmas require membership in the centre of an edge stabilizer. Later it chooses a product of two such labels by its fixed vertices and declares it central with a particular two-edge support. Fixing those vertices does not imply centrality, and the labels need not form the symmetric pair that would give it. Several other fixed-arc shapes remain possible. The asserted configuration is therefore selected without proving that an element with the required properties exists.

Resolution. The replacement works inside an actual Klein four-group of central involutions of the edge stabilizer. It classifies the fixed arcs of its three nonidentity elements using the established rank and parity bounds, and thereby produces an element with the exact support needed for Fan's final commutator argument. The alternative track configuration is treated separately using the group generated by the five local labels. Both cases are proved and combined to establish (5.14), so the proof uses only elements for which perfect position is defined and no longer infers centrality from a fixed-vertex condition.

Case 92. The double alternating cover is not excluded by the displayed generators

Literature. Lemma 3.8, symmetric case [18].

Point of departure. The generated perfect subgroup has alternating quotient, but that does not make it the required block: modulo its natural module it may still be $2 \cdot A_{2m+1}$. Lifts of the displayed 3-cycles do not exclude this central extension. The example built from a natural module and $2 \cdot S_{2m+1}$ shows why the block equality $[O_2(E), E] = O_2(E)$ needs an additional argument. Such an example can fail the paper’s characteristic-subgroup hypothesis, so it is not a counterexample to the lemma; the point is that this decisive hypothesis has not been used at the disputed step.

Resolution. The replacement argument proves the symmetric case with the induction hypothesis available in the minimal-counterexample application. Its difficult five-point cover case is settled by a rigidity argument: count the maximal elementary abelian subgroups containing an element and form a subgroup by a multiplicity threshold. This subgroup is characteristic in the relevant Baumann subgroup, contains the natural commutator module and lies in the action kernel. Both the component and the Sylow subgroup normalize it, contradicting the required properness of $C^*(G, T)$. This excludes the residual double cover in the actual application without assuming that alternating generation alone excludes it. Only the induction-relative version used by the main theorem is proved; the broader standalone cover-exclusion statement is not claimed.

Case 93. The $SL_2(5)$ cover obstructs the block commutator equality

Literature. Lemma 3.8, linear case $n = 2$ [18].

Point of departure. The cited pushing-up result describes the group modulo a module but does not eliminate a central 2-subgroup outside the commutator module. To obtain a block one needs $[O_2(E), E] = O_2(E)$. For $SL_2(2^n)$ with $n \geq 3$, multiplier triviality supplies the missing conclusion. At $n = 2$, however, $SL_2(4) \cong A_5$ has the double cover $SL_2(5)$. The semidirect product of its natural four-dimensional binary module with that cover illustrates the obstruction. Excluding it requires the local characteristic-subgroup hypotheses, which the printed argument does not check at this point.

Resolution. The replacement separates the multiplier-trivial range from the field-four case. It proves the even-characteristic multiplier statement for the larger fields and supplies a distinct theorem excluding the perfect central-two obstruction when $n = 2$. That theorem uses the full conclusion of Campbell’s pushing-up result together with transfer, rather than asserting that the Schur multiplier vanishes. Together, these results establish the block commutator equality in the local $C(G, T)$ argument. This closes a genuine exceptional case while keeping the double cover as a possibility until the extra local argument rules it out.

Case 94. The fixed space in a sum of moved spaces is larger than asserted

Literature. (4.19), Case A of the alternating-group argument [5].

Point of departure. From $U(A) = U(t) \oplus U(t^h)$, the proof asserts $C_{U(A)}(t) = U(t)$. In fact the fixed space also contains $C_{U(t^h)}(t)$, and the available hypotheses do not show that this term vanishes. Consequently a nonzero fixed vector in another moved space need not lie in the intersection used for the contradiction. The objection is about the claimed inference between these actual subspaces in the module, not about the theorem's conclusion. Merely correcting the conjugating-letter misprint in the displayed direct sum does not supply the missing vanishing assertion.

Resolution. The replacement proves an orthogonality statement for distinct commuting conjugates of t : their moved spaces have zero intersection and each involution fixes the other's moved space. Working with the natural alternating-module basis then forces the induced permutation to be a transposition and the involution action to be a transvection. This gives the required odd-transposition property by a different argument. The resulting embedding theorem is used later in the local $C(G, T)$ argument. The claim concerns this repaired branch; it does not make every original intermediate fixed-space equality true.

Case 95. Quaternion normal subgroups in the centralizer analysis

Literature. Theorem 4.1(3), four-transposition case, and Lemma 5.7 [36].

Point of departure. When the permutation involution moves eight points as four transpositions, its centralizer has two normal regular quaternion subgroups with that involution as their unique involution. These groups are omitted from the normal-subgroup clause of Theorem 4.1(3); with three fixed points their products with the odd normal subgroup must also be considered. The proof's module argument controls subgroups inside the elementary abelian part but not these quaternion groups projecting onto a normal four-group. The defect matters because Lemma 5.7 uses the clause in precisely the surviving range $8 \leq n \leq 11$.

Resolution. The corrected normal-subgroup theorem is proved only in the range where the permutation action eliminates the quaternion exception. The four-transposition range is handled separately: normalize the involution to a fixed permutation, classify the relevant centralizer configurations, and use the actual surrounding hypotheses to exclude the unwanted images. This finite check supplies Lemma 5.7. Thus the downstream lemma is recovered without asserting the false universal small-range clause. The finite calculation addresses only the quaternion exception. The other preliminary result required by Lemma 5.7 is separate from that calculation.

Case 96. A unique centralizing involution does not make a component standard

Literature. Lemma 3.1, implications (2) $\Rightarrow$ (1), (3) $\Rightarrow$ (1), and Lemma 3.3 [9].

Point of departure. In $PSp_4(q)$ for odd $q \geq 5$, the two plane factors give commuting conjugate $SL_2(q)$ subgroups. One factor is a component at the central involution of the central product, and its centralizer has only that involution, but it is not standard because the other factor is a distinct commuting conjugate. Thus the general implications asserted in Lemma 3.1 are false. The paper's special family may exclude this example, but that exclusion must be proved; it cannot be imported from a criterion stated for arbitrary components.

Resolution. The corrected criterion explicitly assumes that the component does not commute with any distinct conjugate. The application is split according to whether the component contains the small local subgroups that establish this condition directly. For the remaining L_2 cases, a separate component-theorem argument is supplied within the actual Section 3 setting. Lemma 3.3 follows from these two cases and the corrected criterion. This retains the original ambient hypotheses and proves the additional property where needed, rather than claiming that uniqueness of a centralizing involution already implies standardness.

References

- [1] J. L. Alperin, Richard Brauer, Daniel Gorenstein. *Finite groups with quasi-dihedral and wreathed Sylow 2-subgroups*. Trans. Am. Math. Soc. **151** (1970), 1–261. [DOI](#).
- [2] Michael Aschbacher. *Tightly embedded subgroups of finite groups*. J. Algebra **42** (1976), 85–101. [DOI](#).
- [3] Michael Aschbacher. *A characterization of Chevalley groups over fields of odd order. I*. The Annals of Mathematics **106**, no. 2 (1977), 353–398. [DOI](#).
- [4] Michael Aschbacher. *A characterization of Chevalley groups over fields of odd order. II*. The Annals of Mathematics **106**, no. 3 (1977), 399–468. [DOI](#).
- [5] Michael Aschbacher. *A factorization theorem for 2-constrained groups*. Proc. Lond. Math. Soc. (3) **43** (1981), 450–477. [DOI](#).
- [6] Michael Aschbacher. *Finite groups of rank 3.I*. Invent. Math. **63** (1981), 357–402. [DOI](#).
- [7] Michael Aschbacher. *Some results on pushing up in finite groups*. Math. Z. **177** (1981), 61–80. [DOI](#).
- [8] Michael Aschbacher. *Weak closure in finite groups of even characteristic*. J. Algebra **70** (1981), 561–627. [DOI](#).
- [9] Michael Aschbacher. *A characterization of some finite groups of characteristic 3*. J. Algebra **76** (1982), 400–441. [DOI](#).
- [10] Michael Aschbacher. *The Tits group as a standard subgroup*. Math. Z. **181** (1982), 229–252. [DOI](#).
- [11] Michael Aschbacher. *Finite groups of rank 3. II*. Invent. Math. **71** (1983), 51–163. [DOI](#).
- [12] Michael Aschbacher. *The uniqueness case for finite groups. II*. Ann. Math. (2) **117** (1983), 455–551. [DOI](#).
- [13] Michael Aschbacher, Yoav Segev. *Extending morphisms of groups and graphs*. Ann. Math. (2) **135**, no. 2 (1992), 297–323. [DOI](#).
- [14] Michael Aschbacher, Gary M. Seitz. *Involutions in Chevalley groups over fields of even order*. Nagoya Math. J. **63** (1976), 1–91. [DOI](#).
- [15] Michael Aschbacher, Gary M. Seitz. *On groups with a standard component of known type*. Osaka J. Math. **13** (1976), 439–482. [Source](#).
- [16] Michael Aschbacher, Stephen D. Smith. *The classification of quasithin groups. I: Structure of strongly quasithin K -groups*. Math. Surv. Monogr. 111, 2004. [DOI](#).
- [17] Michael Aschbacher, Stephen D. Smith. *The classification of quasithin groups. II: Main theorems: the classification of simple QTKE-groups*. Math. Surv. Monogr. 112, 2004. [DOI](#).
- [18] D. Bundy, N. Hebbinghaus, B. Stellmacher. *The local $C(G, T)$ theorem*. J. Algebra **300**, no. 2 (2006), 741–789. [DOI](#).
- [19] Axiom Math. *The Unbalanced Group Theorem*. Hexagon preprint 2610.00575v1, 27 October 2026. [Public report](#).

- [20] Alice Devillers, Ralf Gramlich, Bernhard Mühlherr. *The sphericity of the complex of non-degenerate subspaces*. J. Lond. Math. Soc., II. Ser. **79**, no. 3 (2009), 684–700. The detailed finding uses numbering and hypotheses in [arXiv:0711.3468v3](#); final journal concordance was not independently checked. [DOI](#).
- [21] Paul S. Fan. *Amalgams of prime index*. J. Algebra **98** (1986), 375–421. [DOI](#).
- [22] Franz J Fritz. *On centralizers of involutions with components of 2-rank two, II*. Journal of Algebra **47**, no. 2 (1977), 375–399. [DOI](#).
- [23] Robert Gilman, Ronald Solomon. *Finite groups with small unbalancing 2-components*. Pac. J. Math. **83** (1979), 55–106. [DOI](#).
- [24] Daniel Gorenstein, Richard Lyons. *The local structure of finite groups of characteristic 2 type*. Mem. Am. Math. Soc. 42, no. 276, 1983. [DOI](#).
- [25] Daniel Gorenstein, Richard Lyons, Ronald Solomon. *The classification of the finite simple groups. Number 6. Part IV: The special odd case*. Math. Surv. Monogr. 40.6, 2005. [DOI](#).
- [26] Robert M. Guralnick, Gunter Malle. *Classification of 2F-modules. II*. In *Finite Groups 2003*, 117–184, 2004. The detailed finding concerns the archived author manuscript, pp. 9–10; concordance with the final typesetting was not independently checked. [DOI](#).
- [27] Morton E. Harris. *Finite groups containing an intrinsic 2-component of Chevalley type over a field of odd order*. Trans. Am. Math. Soc. **272** (1982), 1–65. [DOI](#).
- [28] Jianhua Huang, Bernd Stellmacher, Gernot Stroth. *Some parabolic systems of rank 2 related to sporadic groups*. J. Algebra **102** (1986), 78–118. [DOI](#).
- [29] A. A. Makhnev. *Finite simple groups with a standard subgroup of the type $L_3(4)$* . Mat. Zametki **37**, no. 1 (1985), 7–12. English translation: Math. Notes 37 (1985), 5–7, DOI [10.1007/BF01652506](#). [Source](#).
- [30] Patrick Paschal McBride. *Near solvable signalizer functors on finite groups*. J. Algebra **78** (1982), 181–214. [DOI](#).
- [31] Ulrich Meierfrankenfeld, Bernd Stellmacher. *Pushing up weak BN-pairs of rank two*. Communications in Algebra **21**, no. 3 (1993), 825–934. [DOI](#).
- [32] Robert H. Gilman, Robert L. Griess Jr.. *Finite groups with standard components of Lie type over fields of characteristic two*. J. Algebra **80** (1983), 383–516. [DOI](#).
- [33] Robert L. Griess Jr., David R. Mason, Gary M. Seitz. *Bender groups as standard subgroups*. Trans. Am. Math. Soc. **238** (1978), 179–211. [DOI](#).
- [34] Robert L. Griess Jr., Ronald Solomon. *Finite groups with unbalancing 2-components of $\{\hat{L}_3(4), He\}$ -type*. J. Algebra **60** (1979), 96–125. [DOI](#).
- [35] Stephen D. Smith. *Large extraspecial subgroups of widths 4 and 6*. J. Algebra **58** (1979), 251–281. [DOI](#).
- [36] Ronald Solomon. *Finite groups with intrinsic 2-components of type $\hat{A}_n$* . J. Algebra **33** (1975), 498–522. [DOI](#).
- [37] Ronald Solomon. *Maximal 2-components in finite groups*. Communications in Algebra **4**, no. 6 (1976), 561–594. [DOI](#).

- [38] Ronald Solomon. *Standard components of alternating type, II*. Journal of Algebra **47**, no. 1 (1977), 162–179. [DOI](#).
- [39] Ronald Solomon. *2-Signalizers in finite groups of alternating type*. Communications in Algebra **6**, no. 6 (1978), 529–549. [DOI](#).
- [40] Ronald Solomon, S. K. Wong. *On $L_2(2^n)$ -blocks*. Proc. Lond. Math. Soc. (3) **43** (1981), 499–519. [DOI](#).
- [41] G. Stroth. *A characterization of Fischer’s sporadic simple group of the order $2^{41} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 31 \cdot 47$* . J. Algebra **40** (1976), 499–531. [DOI](#).
- [42] F. G. Timmesfeld. *Groups with weakly closed TI-subgroups*. Math. Z. **143** (1975), 243–278. [DOI](#).
- [43] Franz Timmesfeld. *Finite simple groups in which the generalized Fitting group of the centralizer of some involution is extraspecial*. Ann. Math. (2) **107** (1978), 297–369. [DOI](#).
- [44] Franz Timmesfeld. *A note on 2-groups of $GF(2^n)$ -type*. Arch. Math. **32** (1979), 101–108. [DOI](#).
- [45] Franz Timmesfeld. *Correction to “Finite simple groups in which the generalized Fitting group of the centralizer of some involution is extraspecial”*. Ann. Math. (2) **109** (1979), 413–414. [DOI](#).
- [46] John H. Walter. *Finite groups with Abelian Sylow 2-subgroups of order 8*. Invent. Math. **2** (1967), 332–376. [DOI](#).
- [47] John H. Walter. *The $\mathfrak{B}$ -conjecture; characterization of Chevalley groups*. Mem. Am. Math. Soc. 61, no. 345, 1986. [DOI](#).